

The Drazin index and minimal polynomials of special sums

Faustino Maciala¹, C. Mendes Araújo², P. Patrício²

¹CMAT – Centre of Mathematics, Universidade do Minho, Portugal
Departamento de Ciências da Natureza e Ciências Exatas do Instituto Superior
de Ciências da Educação de Cabinda, Angola

fausmacialamath@hotmail.com

²CMAT – Centre of Mathematics and Department of Mathematics,
Universidade do Minho, Portugal

clmendes@math.uminho.pt, pedro@math.uminho.pt

Abstract.

For a singular matrix A and any von Neumann inverse $A^- \in A\{1\}$, we study the matrix A^2A^- and the sum $A^2A^- + I - AA^-$. We prove that their minimal polynomials and Drazin indices are independent of the choice of A^- , and we obtain the explicit formulas

$$\psi_A(\lambda) = \lambda \psi_{A^2A^-}(\lambda), \quad \psi_{A^2A^- + I - AA^-}(\lambda) = \text{lcm}\{\lambda^{-1}\psi_A(\lambda), \lambda - 1\},$$

which yield $i(A^2A^- + I - AA^-) = i(A) - 1$. Thus this transformation reduces the Drazin index of A by exactly one, in a way fully determined by ψ_A and independent of the generalized inverse used. This controlled index reduction is the key technical tool behind our analysis of anti-triangular block matrices $M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$, yielding sharp bounds for $i(M)$ in terms of $i(A)$ and $i(BC)$.

Keywords: Drazin inverse; von Neumann inverse; minimal polynomial; Drazin index; anti-triangular block matrix.

Acknowledgements. This research was partially financed by Portuguese Funds through FCT (Fundação para a Ciência e a Tecnologia) within the Project UID/00013/2025. <https://doi.org/10.54499/UID/00013/2025>

References

- [1] M. P. Drazin, *Pseudo-inverses in associative rings and semigroups*, American Mathematical Monthly, **65**: 506–514 (1958).
- [2] R. E. Hartwig and J. M. Shoaif, *Group inverses and Drazin inverses of bidiagonal and triangular Toeplitz matrices*, Journal of the Australian Mathematical Society Series A, **24**: 1–34 (1977).
- [3] C. D. Meyer and N. J. Rose, *The index and Drazin inverse of block triangular matrices*, SIAM Journal on Applied Mathematics, **33**: 1–7 (1977).
- [4] P. Patrício and R. E. Hartwig, *The $(2,2,0)$ group inverse problem*, Applied Mathematics and Computation, **217**: 2–11 (2010).