Fractional Calculus

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Abstract

In 1695 L'Hôpital wrote a letter to Leibniz asking for the meaning of $\mathrm{D}^n y$ for $n = 1/2$. Leibniz replied 'It seems that useful consequences shall be drawn from these paradoxes one day, as there are no paradoxes that do not prove useful'. The term 'Fractional Calculus' (FC) was adopted at that time and is used even nowadays, although many researchers find more adequate the terminology 'integration and differentiation of arbitrary order'. Starting with the ideas of Leibniz many important mathematicians developed the theoretical concepts, but practical aspects were not evident. During the thirties A. Gemant and O. Heaviside applied FC in the areas of mechanical and electrical engineering, respectively. Nevertheless, these important contributions were somehow forgotten and only during the eighties, we find relevant work, by A. Oustaloup, that developed a pioneering work in the FC application in automatic control systems. In the same period, FC emerged as an important tool associated with phenomena such as fractal and chaos and, consequently, in the modelling of dynamical systems. The ongoing research of FC application addresses many different aspects such as viscoelasticity and damping, biology, electronics, signal processing, system identification, diffusion and wave propagation, percolation, modeling, identification, and control. Bearing these ideas in mind, this lecture introduces the FC fundamental mathematical aspects and discuses some of their consequences. Based on the FC concepts, the lecture reviews the main approaches for implementing fractional operators and discusses the adoption of FC in control systems. Finally are presented some applications in the areas of modelling and control, namely fractional PID, heat diffusion systems, electromagnetism, fractional electrical impedances, evolutionary algorithms, robotics, and nonlinear system control.

Keywords: Fractional calculus; modelling; simulation.

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