Golden Tilings

A. Pinto[1], J. P. Almeida[2], A. Portela[3]

[1]Univ. do Minho, [2]CMAT-FCT da U. Minho e I. P. Bragança, [3]Univ. de Montevideo.

A. Pinto and D. Sullivan [3] proved a one-to-one correspondence between: (i) C^{1+} conjugacy classes of expanding circle maps; (ii) solenoid functions and (iii) Pinto-Sullivan's dyadic tilings on the real line. Here, we prove a one-to-one correspondence between: (i) golden tilings; (ii) smooth conjugacy classes of golden diffeomorphism of the circle that are fixed points of renormalization; (iii) smooth conjugacy classes of Anosov difeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugated to the Anosov automorphism $G(x,y) = (x+y, x)$ and (iv) solenoid functions.

The solenoid functions give a parametrization of the infinite dimensional space consisting of the mathematical objects described in the above equivalences. In this case, the expanding dynamics are hidden in the renormalization operator that acts on the minimal set. The link between Anosov diffeomorphisms and diffeomorphisms of the circle, that are smooth fixed points of renormalization, is due to D. Sullivan and E. Ghys. The renormalization operator appears inspired in the works of Feigenbaum and Lanford. Pinto-Rand [2] proved the equivalence between (i) and (ii). Here, we present the renormalization in a new way, using the construction of a train-track, as an intermediate step (see also Pinto-Rand [2]). The train-track appears as in the works of Thurston, Penner, Williams and Veech, but with a new and relevant feature that corresponds to have a C^{1+} structure associated to it. Here we explicit the definition of *golden tilings*. The properties of the golden tilings are described using a Fibonacci decomposition for the natural numbers.

References and Literature for Further Reading

[1] A. A. Pinto, J. P. Almeida, A. Portela, Golden tilings, *submitted*.

[2] A. A. Pinto, D. Rand, Renormalisation gives all surface Anosov diffeomorphisms with a smooth invariant measure, *submitted*.

[3] A. A. Pinto, D. Sullivan, The circle and the solenoid, Dedicated to A. Katok on the occasion of his 60th birthday, DCDS-A, **16** (2), 463-504, (2006).