First cohomology group and invariant distributions

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Abstract: Let $\Gamma\colon \mathbb{G} \times M \to M$ be a smooth action of the Lie group $\mathbb{G}$ on a closed differentiable manifold M. A map $A\colon \mathbb{G} \times M \to \mathbb{R}$ is said to be a *cocycle* over Γ when it satisfies

$$A(hg, x) = A(h, \Gamma(g, x)) + A(g, x), \quad \forall g, h \in \mathbb{G},\ \forall x \in M.$$

On the other hand, a cocycle A is called a *coboundary* if there exists $u\colon M \to \mathbb{R}$ verifying

$$A(g, x) = u(\Gamma(g, x)) - u(x), \quad \forall g \in \mathbb{G},\ \forall x \in M.$$

This kind of objects appears very naturally in different contexts in dynamics. Indeed, a lot of questions about the dynamical properties of Γ, especially those on rigidity, can be reduced to determine whether a given cocycle is a coboundary.

In this lecture we shall discuss several aspects of this problem and talk about the necessity of studying the space of Γ-invariant distributions (in Schwartz' sense) when we are trying to determine the structure of the *first cohomology group* of Γ, i.e. the quotient space of cocycles modulo coboundaries.

Finally, we will talk about *cohomological rigidity*, presenting some examples and discussing some recent results about classification of cohomologically rigid $\mathbb{R}$- and $\mathbb{Z}$-actions on low dimensional manifolds.