

# Stable equilibria in moment closure and stochastically quasi-stable equilibria in SIS model

José Martins <sup>\*</sup>, Alberto Pinto & Nico Stollenwerk

<sup>\*</sup> Polytechnic Institute of Leiria, School of Technology and Management, Leiria  
University of Minho, Braga

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## Abstract

In this work we study for the SIS model, also known as the birth-death process, the dynamics of the higher moments of infected. From the master equation of the process, we deduce the dynamic equations for the moments and to close these equations we use a recursive process that consists in applying the moment closure technique to approximate the higher moments by lower ones. The first approximation, and the simplest, is the mean field approximation. Under this approximation, the model exhibits a poor behavior and it is not satisfactory. Considering two moments, we use the Gaussian approximation and we observe some unbiological regions that can be understood as limitations of the model. Considering more moments, we observe the critical value of the infection rate  $\beta$  increasing and computing the stationary states, in the moment closure approximation, we approach the quasi-stationary states of the SIS model.

**Keywords:** Stochastic processes; SIS model; Moment closure; Quasi-stationary states.

## References

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