Bumps, breathers, and waves in a neural network with threshold accommodation

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Abstract

Many of the equations describing the dynamics of neural systems are written in terms of firing rate functions, which themselves are often taken to be threshold functions of synaptic activity. Dating back to work by Hill in 1936 it has been recognized that more realistic models of neural tissue can be obtained with the introduction of state-dependent dynamic thresholds. In this talk I will discuss a specific phenomenological model of threshold accommodation that mimics many of the properties originally described by Hill. Importantly I will show how to explore the consequences of this dynamic threshold at the tissue level, by modifying a standard neural field model of Wilson-Cowan type. As in the case without threshold accommodation classical Mexican-Hat connectivity is shown to allow for the existence of spatially localized states (bumps) in both one and two dimensions. Importantly an analysis of bump stability in one dimension, using recent Evans function techniques, shows that bumps may undergo instabilities leading to the emergence of both breathers and traveling waves. Moreover, a similar analysis for traveling pulses leads to the conditions necessary to observe a stable traveling breather. In the regime where a bump solution does not exist direct numerical simulations show the possibility of self-replicating bumps via a form of bump splitting. Simulations in two space dimensions show analogous localized and traveling solutions to those seen in one dimension. Indeed dynamical behavior in this neural model appears reminiscent of that seen in other dissipative systems that support localized structures, and in particular those of coupled cubic complex Ginzburg-Landau equations. Further numerical explorations illustrate that the traveling pulses in this model exhibit particle like properties, similar to those of dispersive solitons observed in some three component reaction-diffusion systems.

Keywords: neural field models; pattern formation;

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