Dynamics on spectral solutions of forced Burgers equation

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Abstract

Burgers equation $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \delta\frac{\partial^2 u}{\partial x^2} + f(x)$ is one of the simplest partial nonlinear differential equation which can develop discontinuities, being the driven equation used to explore unidimensional 'turbulence'. The higher points of the nonlinear waves travel at a higher speed and shocks and discontinuities for low values of δ will tend to appear in the intervals where u is decreasing. Phenomena as wave processes, traffic flow, shocks, acoustic transmission and gas dynamics can be studied starting from this equation. For low values of the viscosity coefficient δ, by discretization through spectral collocation methods, oscillations in Burgers equation can occur. For the Dirichlet problem and under a dynamic point of view, several bifurcations and stable attractors can be observed. Periodic orbits, quasiperiodic and strange ones may arise. Bistability can also be observed. Numerical simulations indicate that the loss of stability of the asymptotic solution of Burgers equation must occur by means of a supercritical Hopf bifurcation.

Many nonlinear phenomena are modeled by spatiotemporal systems of infinite or very high dimension. Coupling and synchronization of spatially extended dynamical systems, periodic or chaotic, have many applications, including communications systems, chaos control, estimation of model parameters and model identifications. For the unidirectionally coupling, numerical studies show the presence of identical and generalized synchronization for different values of spacial points and different values of the viscosity coefficient δ. For u representing the drive variable, v the driven one and α the coupling parameter, nonlinear coupling at the waves velocity v, obtained by replacing the discretized response variable v by $u + \alpha(u - v)$, with $0 < \alpha < 1$, identical or generalized synchronization is achieved, allowing only values of α around 1 in very few cases. This points out the fact that although the partial replacement may not reach synchronization, nonlinear coupling with $0 < \alpha < 1$ may do it.

Keywords: Dynamical systems, Burgers equation, Spectral methods, Synchronization

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