

Dynamics of Populational Growth Models with Allee Effect

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Abstract

In this work, we consider populational growth models with Allee effect. These models are proportional to beta densities with shape parameters p and 2, where the dynamical complexity is related with the malthusean parameter r . For $p > 2$, these models exhibit a population dynamics with natural Allee effect. However, in the case of $1 < p \leq 2$, the proposed models do not include this effect. In order to invoke the Allee effect, we present some alternative mechanisms and investigate their dynamics. Using dynamical symbolic techniques, we analyse the complex behaviour of these models, in terms of topological entropy, in the parameter plane (r, p) , defining different dynamical regimes.

References

- [1] S. M. Aleixo, J. L. Rocha and D. D. Pestana, Populational Growth Models in the Light of Symbolic Dynamics (in press), *ITI2008 Conference Proceedings*, (accepted in May of 2008).
- [2] D. S. Boukal and L. Berec, Single-species Models of the Allee Effect: Extinction Boundaries, Sex Ratios and Mate Encounters, *J. Theor. Biol.*, **218**, (2002), 375 — 394.
- [3] S. Brianzoni, C. Mammana and E. Michetti, Nonlinear Dynamics in a business-cycle model with logistic population growth, *Chaos, Solitons and Fractals*, (2007), doi:10.1016/j.chaos.2007.08.041
- [4] R. S. Cantrell and C. Cosner, Density dependent behavior at habitat boundaries and the Allee effect, *Bull. Math. Biol.*, **69**, (2007), 2339 — 2360.
- [5] F. M. Hilker, M. Langlais, S. Petrovskii and H. Malchow, A diffusive SI model with Allee effect and application to FIV, *Math. Biosci.*, **206**, (2007), 61 — 80.
- [6] R. Lopez-Ruiz and D. Fournier-Prunaret, Indirect Allee effect, bistability and chaotic oscillations in a predator-prey discrete model of logistic type, *Chaos, Solitons and Fractals*, **24**, (2005), 85 — 101.
- [7] S. P. Rogovchenko and Y. V. Rogovchenko, Effect of periodic environmental fluctuations on the Pearl-Verhulst model, *Chaos, Solitons and Fractals*, (2007), doi:10.1016/j.chaos.2007.11.002
- [8] D. Ruelle, *Thermodynamic Formalism*, Addison-Wesley, Reading, MA., 1978.
- [9] S. J. Schreiber, Allee effects, extinctions and chaotic transients in simple population models, *Theor. Pop. Biol.*, **64**, (2003), 201 — 209.
- [10] P. Walters, *An Introduction to Ergodic Theory*, Graduate Texts in Math., **79**, Springer-Verlag, 1981.

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