Days in Logic 2018

Manuel António Martins, Isabel Oitavem, Luís Pinto
Editors

Welcome

Days in Logic is a biennial meeting that aims at bringing together mathematicians, computer scientists and other researchers from Portugal, and elsewhere, with interest in Logic. It is specially directed to graduate students. Along the years Days in Logic has brought to Portugal some of the most prominent researchers in the area of Logic.

Days in Logic takes place since 2004, and has always combined keynote courses, by invited speakers, with contributed talks. In this 8th edition of Days in Logic, there will be 3 invited courses, in the areas of implicit computational complexity, duality, and hybrid logic, and 19 contributed talks, in various areas of mathematical logic, philosophical logic, and logic in computer science.

The organization of the meeting was financed by CIDMA, Universidade de Aveiro, through project UID/MAT/04106/2013, and by CMA, Universidade Nova de Lisboa, through project UID/MAT/00297/2013, and was locally supported by Daniel Figueiredo (CIDMA) and Diana Costa (CIDMA).

The organizers of Days in Logic 2018 welcome and whish a very fruitful and pleasent meeting to all participants of the meeting.

20 January 2018,

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Contents

Abstracts of invited courses

From linear logic to types for implicit computational complexity

Patrick Baillot

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Linear logic provides a fine-grained logical system to study the dynamics of computation in lambda-calculus. In particular it gives a logical status to the duplication of arguments, thanks to a specific modality. This has triggered logical approaches to implicit computational complexity, a research field aiming at characterizing complexity classes without refering to resource bounds. In this course we will present a guided tour of the contributions of linear logic to implicit computational complexity. We will try to highlight the key ideas of Elementary and Light linear logic through the study of proof-nets and their reduction. We will also show how these logics give rise to type systems for lambda-calculus, ensuring that well-typed programs admit certain time complexity bounds.

Three Lectures on Hybrid Logic

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This course introduces hybrid logic, a form of modal logic in which it is possible to name worlds (or times, or computational states, or situations, or nodes in parse trees, or people - indeed, whatever it is that the elements of Kripke Models are taken to represent. The course has three major goals. The first is to convey, as clearly as possible, the ideas and intuitions that have guided the development of hybrid logic. The second is to teach something about hybrid deduction and its completeness theory, and to make clear the crucial role played by the basic hybrid language and the Henkin construction. The third is to give you a glimpse of more powerful hybrid systems beyond the basic language, notably languages using the downarrow binder and explicit quantification over nominals.

Here is the lecture plan:
Lecture 1: From modal logic to hybrid logic
Lecture 2: Hybrid deduction
Lecture 3: Stronger systems

I won't be presuming any particular background in hybrid (or indeed, modal) logic, but I will be assuming a certain "logical maturity". As background reading I would like to suggest the following:

1. "Representation, Reasoning, and Relational Structures: a Hybrid Logic Manifesto", by Patrick Blackburn, Logic Journal of the IGPL, 8(3), 339-625, 2000. **(An easy introduction to the topic)**

2. Section 7.4, Hybrid Logic, pages 434-445 of Modal Logic, by Patrick Blackburn, Maarten de Rijke and Yde Venema. Cambridge Tracts in Theoretical Computer Science, 53, Cambridge University Press, 2001 **(This covers basic completeness theory for the basic language)**

3. "Hybrid Logic", by Carlos Areces and Balder ten Cate, Handbook of Modal Logic, edited by Blackburn, van Benthem and Wolter, 2007, pages 821-868, Elsevier. **(An advanced introduction to the topic)**

4. "Contextual Validity in Hybrid Logic", by Patrick Blackburn and Klaus Frovin Jørgensen., Proceedings of CONTEXT 2013, Lecture Notes in Artificial Intelligence (LNAI) 8175, pages 185-198, 2013. **(An example of more recent work)**

Duality theory

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In [2], the authors make the seemingly paradoxical observation that "... an equation is only interesting or useful to the extent that the two sides are different!". Indeed, a moment's thought convinces us that an equation like $e^{i\omega} = \cos(\omega) + i\sin(\omega)$ is far more interesting than the rather dull statement that $3 = 3$, and a comparable remark applies if we go up in dimension: equivalent categories are thought to be essentially equal, but an equivalence is of more interest if the involved categories look different. Numerous examples of equivalences of "different" categories relate a category X and the dual of a category A; such an equivalence is called a *dual equivalence* or simply a *duality*. As examples we mention here the classical Stone-dualities (see [14,15]) for Boolean algebras respectively distributive lattices, Esakia's duality theorem for Heyting algebras (see [4]), and the duality for Boolean algebras with operator of [9,10,11].

In these lectures we give a broad overview of several techniques and results concerning the study and construction of dual equivalences. We start by succinctly recalling the main ingredients from category theory, and then discuss the structure of dual equivalences and, more generally, of dual adjunctions between concrete categories. Here we will see that, under mild conditions, every such adjunction is induced by a so-called *dualising object*. Starting from the other end, we give sufficient conditions on a dualising object to yield a dual adjunction and illustrate this procedure with various examples. Dual equivalences constructed this way are often called *natural dualities*, for more information we refer to [8,13,3].

Other techniques leading to dual equivalences can be characterised by the slogan "move from models to syntax". For instance, the theory of monads is one of the main category theoretic formulations of universal algebra, and the dual equivalence between monads (syntax) and monadic categories (semantics) is at the heart of the proofs of the classical Gelfand and Pontryagin dualities presented in [12]. We present here a similar argumentation which was employed in [7] to derive in a uniform way several duality theorems involving categories of relations and categories of algebras with "hemimorphisms", generalising this way the approach of [6] to duality theory for Boolean algebras with operators.

Not surprisingly, it is often easier to construct a dual equivalence only between finite objects. In order to extend this equivalence to all objects, another duality between syntax and semantics comes handy, namely the Gabriel-Ulmer duality for locally presentable categories on one side and limit sketches on the other (see [5,1]). If time permits, we sketch this approach and show in particular how it leads to a "two for the prize of one" principle: obtain a new duality from a given one simply by structure interchange.

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Abstracts of contributed talks

Discrete polymorphism with dynamic types

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Gradual typing [3,1] enables dynamic and static typing in the same language, by extending an existing type system with a dynamic (?) type and consistency ($\sim$) relation that compares gradual types. We can choose between disciplines by inserting a dynamic type annotation in dynamically typed parts of the program, dellaying type checking to the evaluation following an operational semantics which explicitly checks types. It is also important to note that polymorphism, both Hindley-Milner and parametric are supported in gradual typing.

The intersection type system [2] is a counterpart to the Hindley-Milner system, regarding polymorphism. By allowing expressions to be typed with a range of different types, the system provides discrete polymorphism. However, this system is more expressive than the Hindley-Milner system, due to being able to type more expressions, self application for example. Intersection types are of the form $T_1 \cap \ldots \cap T_n$, and the type system for intersection types extends the simply typed lambda calculus to deal with intersections.

In this work we extend the intersection type system with gradual typing, resulting in a system that contains all the expressive power of the intersection type system, and all the advantages of gradual typing. In our system, the type ? may be used in the type connective $\cap$, which allows a single expression to be typed with dynamic and static types simultaneously. For example, the expression $(\lambda x : Int \cap ?\ .\ x + x)\ 1$ has type Int, however, the left expression in the application types with an intersection type: $\lambda x : Int \cap ?\ .\ x + x$ types with $Int \cap ? \rightarrow Int$ and 1 types with Int. This type system adheres to some correctness criteria presented in [1], such as conservative extension and monotonicity w.r.t. precision.

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Open versus closed types

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Regular types are common in the definition of datatypes. They denote sets of values in an algebra of program constructors and correspond to tree automata in the sense that a regular type represents the set of terms (trees) accepted by a tree automaton. Automatic inference of regular types has been used as the basis of type inference for several untyped languages. The approaches are usually based on a over-approximation of the program semantics [2, 3] or on the definition of Hindley-Milner style type systems [4]. Most of these type inference algorithms infer types that over-approximate the program semantics, returning as output a supertype, i.e. a type that is a superset of the actual type of the program.

The nature of untyped languages such as Prolog or Erlang makes it sometimes difficult to infer regular types that are meaningful, for instance the Prolog predicate for the concatenation of lists accepts more than just lists, it also accepts the goal $append([\,], 1, 1)$. Because of this, the type inference algorithm's over-approximation does not correspond to the author's intention, since the original program itself is too general.

We divide types into two classes: open and closed. Open types are types that have at least one variable that occurs unconstrained and closed types are types whose variables are all constrained. We argue that closed types are closer to the programmer's intention and they are much more clear, informative and easy to read [1]. Being more restrictive and closer to the programmer's intention they also help on the debugging process in several situations where open types are not useful.

In this talk we will present the notions of open and closed types, show the main intuition behind their definitions, several examples of its application to programming language verification and how they are related to each other.

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Semantics for combined Hilbert calculi

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Combining Hilbert calculi is well known to correspond to the mechanism of fibring logics, yielding the smallest (Tarskian) logic that extends the components. Moreover, Hilbert calculi are notoriously non-modular, which makes their understanding particularly challenging. In this talk, culminating a long research path, we will finally outline the ingredients of a workable semantics for them [1,2]. The results rely on using possibly partial non-deterministic matrices instead of the most common logical matrices, and on the properties of a straightforward but rich saturation operation. Using them, we show how to directly obtain complete semantics for combined Hilbert calculi by suitably combining the semantics of their components. We illustrate the results with some meaningful examples.[1]

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A Herbrand-like theorem for hybrid logic

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The original version of Herbrand's theorem [2] for first-order logic provided the theoretical underpinning for automated theorem proving, by allowing a constructive method for associating with each first-order formula χ a sequence of quantifier-free formulas $\chi_1, \chi_2, \chi_3, \cdots$ so that χ has a first-order proof if and only if some χ_i is a tautology. Some other versions of Herbrand's theorem have been developed for classical logic, such as the one in [3], which states that a set of quantifier-free sentences is satisfiable if and only if it is propositionally satisfiable.

The literature concerning versions of Herbrand's theorem proved in the context of non-classical logics is meager. We aim to present a version of Herbrand's theorem for hybrid logic, which is an extension of modal logic that is expressive enough so as to allow reference to specific states, and to the accessibility relations and equality between states, thus being completely suitable to deal with relational structures [1]. Our main result states that a set of satisfaction statements is satisfiable if and only if it is propositionally satisfiable.

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Fundamental groups in general o-minimal structures and some comparison results

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Working in an arbitrary *o*-minimal structure $M = (\mathrm{M}, <, \ldots)$ with definable choice functions [1] we present the general *o*-minimal fundamental group functor from [4], prove some of its basic properties and address an unproved claim made in [4]. In particular, we show in a direct way that cells are definably path connected and definably simply connected. Several comparison theorems can then follow. The reason for this is the fact that the fundamental group functor presented satisfies the conditions given in the concluding remarks of [3] under which one can prove in exactly the same way all the main results of that paper. Indeed, these conditions play, in the *o*-minimal context, the same role that analogue properties play in topology (with one exception to ensure local definability which is essential in the *o*-minimal context). A paper reporting on this work is in preparation [2].

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Inferential structures in sequent calculi: metainferences

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If we associate a consequence relation to a sequent calculus in the usual fashion—saying that A follows from X iff the sequent $X : A$ is provable in the calculus—then *modus ponens* is the sequent $A, A \to B : B$. There is also a so-called *metainferential* version of *modus ponens*, namely the sequent rule: $\dfrac{X : A, Y \qquad X : A \to B, Y}{X : B, Y}$, which may or may not be admissible in a calculus depending on its structural properties.[2]

Metainferences have recently become a hot topic in philosophical logic, particularly because it looks as though one may lose metainferences without losing inferences [2,3,5]. Here 'losing' means *invalidating*. For instance, a Cut-less version of LK interpreted using Kleene's strong valuations will retain, under a suitable definition of consequence, all the valid sequents of LK but Cut won't be admissible any longer (nor, for that matter, would the above mentioned metainferential *modus ponens*). This is the logic ST defended in [5].

This raises interesting questions pertaining to, e.g., the identity of logics: Is, for instance, ST classical logic? It also raises questions concerning the categorisation of logics: Is a logic paraconsistent if it invalidates only the metainferential *ex falso* but validates the inferential form?

The answers to these questions depend on what these *metainferences* are and on what role they play in our logical theories. This talk will compare two concepts of *metainferential validity*, one *global* [1] and one *local* [4] and argue that it is the local version that is philosophically relevant and philosophically motivated. On this conception of metainferential validity, there is scope for some surprises when answering the questions above.

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[2]For simplicity, I present the rules in their additive guise. Since this talk is about substructural logics, this is suboptimal but necessary for the sake of concision.

Characterization of strong normalizability for a lambda-calculus with co-control

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We study strong normalization in the system $\overline{\lambda\tilde{\mu}}$, a lambda calculus of proof terms with co-control for the intuitionistic sequent calculus [3]. In this sequent lambda calculus, the management of formulas on the left hand side of typing judgements is "dual" to the management of formulas on the right hand side of the typing judgements in Parigot's $\lambda\mu$-calculus [8] - that is why our system has first-class "co-control". In particular, $\overline{\lambda\tilde{\mu}}$ contains the $\tilde{\mu}$-operator, which comes with a reduction rule that triggers a dual concept of structural substitution - the structural substitution of a "co-continuation" $\mathcal{H}$ for a proof variable x. The rule for $\tilde{\mu}$ coexists with four other reduction rules. Together, these rules reduce expressions of $\overline{\lambda\tilde{\mu}}$ to a form corresponding to the cut-free proofs of LJT [6] - hence, the reduction rules express a combination of cut-elimination and focalization [7].

We characterize strong normalizability in $\overline{\lambda\tilde{\mu}}$ as typability in a system for assigning intersection types. The typing system we propose for $\overline{\lambda\tilde{\mu}}$ is obtained by adapting the system used to characterize the strongly normalizing proof terms of $\lambda\mathsf{Gtz}$, in previous work with colleagues [4,5]. The $\lambda\mathsf{Gtz}$-calculus [2] is another sequent lambda calculus, where the treatment of the $\tilde{\mu}$-operator follows the original and simpler one found in [1]: it is a term-substitution former, and its reduction principle triggers an ordinary term substitution. The characterization of strong normalizability in $\overline{\lambda\tilde{\mu}}$ is proved, not by re-running the proof for $\lambda\mathsf{Gtz}$, but by "lifting" the characterization in $\lambda\mathsf{Gtz}$. This requires a detailed comparison of the two rewriting systems, which is of independent interest, as it highlights sensitive choice points in the design of calculi of proof terms for the sequent calculus, particulary the treatment of proof-term variables and the related substitution principles.

Finally, since it is known how to obtain bidirectional natural deduction systems isomorphic to the sequent calculi $\overline{\lambda\tilde{\mu}}$ and $\lambda\mathsf{Gtz}$, characterizations are obtained of the strongly normalizing proof terms of such natural deduction systems, again by turning them into systems for assigning intersection types. In the resulting systems, there is a noteworthy interplay between change of "directionality" and the style of the inference rules for the intersection type former.

This work has been published in [9].

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A herbrandized functional interpretation of classical first-order logic

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We define a (cumulative) functional interpretation of first-order classical logic and show that each theorem of first-order logic is naturally associated with a certain scheme of tautologies. Herbrand's theorem is obtained as a special case. The schemes are given through formulas of a language of finite-type logic defined with the help of an extended typed combinatory calculus that associates to each given type the type of its nonempty finite subsets. New combinators and reductions are defined, the properties of strong normalization and confluence still hold and, in reality, they play a crucial role in defining the above mentioned schemes. The functional interpretation is dubbed "cumulative" because it enjoys a monotonicity property now so characteristic of many recently defined functional interpretations.

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Intervalar differential dynamic logic

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Differential dynamic logic was proposed by A. Platzer [2], is able to reason about hybrid systems by considering two types of atomic programs – discrete assignments and continuous evolutions (described by differential equations).

However, in general, when one works with differential equations, it is known that small changes in the initial conditions can lead to great changes in the continuous evolutions.

This turns to be a relevant issue in real life systems since it is impossible to measure exact values for variables like distance and velocity. Because of this, we propose an intervalar version of differential dynamic logic. In our version, the variables are not interpreted as real values but as closed interval, leading to a methodological representation of uncertainty and experimental error.

For this purpose, we consider an algebra for intervals which, in fact, was already develop by Ramon Moore in the 50's and is described in his PhD thesis [1] and was applied to dynamic logic by R. Santiago *et al.* [3]. We intend to exploit this concept and extend it to differential equations. If possible, we would like to obtain a proof calculus for this intervalar arithmetics.

Furthermore, the interpretation of variables as intervals can cause some problems when one is trying to evaluate a formula in a Boolean way. For instance, it seems that there exists a graduated trueness for the proposition $x = [0, 6]$ because it seems to be "less false" if x is $[0, 5]$ rather than $[0, 2]$. An option could be to consider membership degrees, *i.e.*, assert that $x \in [a, b]$ with a truth value on $[0, 1]$.

In this way, it seems reasonable to introduce a fuzzy measure in the semantics. Here, three hypothesis arise: i) either to consider fuzziness only in the interpretation of the formulas; ii) to consider fuzzy modalities or iii) to consider membership degrees for variables.

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Towards Hoare logic for reasoning about weighted computation

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Hoare logic was the first formal system for verification of classic imperative programs. The main ideas of Hoare logic center on a syntax to reason about Partial Correctness Assertion (PCA) and a deductive system with specialised rules of inference. In [1], Dexter Kozen shows that it is possible to replace the deductive apparatus of Hoare logic by simple equational reasoning, using Kleene algebra with tests (KAT). This algebraic structure is taken as the standard algebra to model and reason about classic imperative programs, i.e. sequences of discrete actions guarded by Boolean tests. This work tries to discuss Propositional Hoare logic (PHL) for weighted computations: programs as weighted transitions and tests with outcomes in a not necessary bivalent truth space. In order to express these types of computations, we introduce two generalisations of KAT, namely graded Kleene algebra with tests (GKAT) and Heyting Kleene algebra with tests (HKAT), in which we will encode PHL.

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Boolean tableaux for abstract argumentation frameworks

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A tableaux method is proposed as a decision procedure for Dung's argumentation frameworks. Argumentation frameworks are structures (A, R), where A is a set of arguments and R is a binary ("attack") relation over A. The method enables to decide the justification of sentences p such as 'argument a is accepted' or 'argument a is rejected', and captures credulous and skeptic behaviors for both preferred and grounded semantics. To show this, we introduce notions of satisfiability (the tableaux of p has an open branch) and validity (p is satisfiable and its tableaux is free of loops). Moreover, the method is defended as a useful tool for teaching on semantics for argumentation frameworks, given the simplicity and familiarity of logicians with analytic tableaux.

Key words: argumentation frameworks; tableaux methods; extension semántics

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Towards the dynamic logic with binders $\mathcal{D}^{\downarrow}$

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We discuss in this talk our developments in $\mathcal{D}^{\downarrow}$ [1], a new dynamic logic that combines regular modalities with the binder constructor typical of hybrid logic. This logic provides a smooth framework for the stepwise development of reactive systems. Actually, the logic is able to capture system properties at different levels of abstraction, from high-level safety and liveness requirements, to constructive specifications representing concrete processes.

The model class semantics of specifications in $\mathcal{D}^{\downarrow}$ is, however, not closed under bisimulation equivalence. We discuss our recent developments in [2,3] where we define an observational semantics for $\mathcal{D}^{\downarrow}$. This involves the definition of a new model category and of a more relaxed satisfaction relation.

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Modular analysis of Hilbert calculi

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Culminating a long research path, we have recently identified the interaction patterns of combined Hilbert calculi, and established the ingredients of a workable modular semantics for them. In this presentation we shall give an overview of these contributions by means of illustrating examples.

Namely, in [3], we have played a game where we depart from different Hilbert calculi given by subsets of rules for classical implication ($\to$) and bottom ($\neg$), and study the negations defined by the usual abbreviation ($\neg A := A \to \bot$) in each of the given logics. In each case we extract a semantics for the defined $\to, \bot$-logics using the general recipes for fibred logics [2,5], and then also for the corresponding $\neg$-only fragment. Using [1] we further obtain upper bounds for the complexity associated to deciding these logics.

In a distinct setting, in [4], we take advantage of the same technical tools and of Post's classification in order to show that classical logic cannot be broken into two disjoint non-functionally complete fragments (except in very extreme circumstances). Using the general recipe for fibring we can now give semantics to the myriad logics obtained by combining different fragments of classical logic.[3]

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From complementary logic to proof-theoretic semantics

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Two deductive systems $\mathscr{S}$ and $\overline{\mathscr{S}}$, sharing a same language, are said to be *complementary* when:

$$\vdash_{\overline{\mathscr{S}}} \varphi \text{ if, and only if, } \nvdash_{\mathscr{S}} \varphi.$$

In other words, a system $\overline{\mathscr{S}}$ turns out to be complementary with respect to another system $\mathscr{S}$ if it proves exactly the non-theorems of $\mathscr{S}$ [3,10,8]. The conceptual idea underlying the study of logical complementarity is that of the characterization of a decidable system $\mathscr{S}$ by taking, so to speak, its picture in the negative. The term 'characterization' has here a precise meaning in the sense that theorems of the positive part $\mathscr{S}$ can be ascertained by excluding the possibility of their provability in the complementary system $\overline{\mathscr{S}}$. As a matter of fact, logical complementarity should be thought of as a way to sharpen our proof-theoretical understanding of decidable calculi to the extent that it allows us to widen the space of proofs so as to include complementary derivations. This is in line with Prawitz's idea of a "general proof theory" according to which "proofs are studied in their own right in the hope of understanding their nature" [4].

In the first part of my talk, I shall be concerned with $\overline{LK}$, a cut-free sequent calculus able to faithfully characterize classical (propositional) non-theorems [9,2]. I will show how to enrich $\overline{LK}$ with two admissible (unary) cut rules, which allow for a simple and efficient cut-elimination algorithm. Two relevant facts will be underlined: (*i*) complementary cut-elimination always returns the simplest proof for any given provable sequent, and (*ii*) provable complementary sequents turn out to be "deductively polarized" by the empty sequent, in the sense that any $\overline{LK}$ proof can be seen as a subproof of a longer proof ending with the empty sequent [1].

In the second part, I will observe how an alternative sequent system for complementary classical logic can be devised by slightly modifying Kleene's system G4 [6]. This move is meant to pave the way for a new kind of proof-theoretic semantics for classical logic [7].

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Probabilistic logic of quantum observations

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A probabilistic propositional logic, endowed with an epistemic component for asserting (non)-compatibility of diagonizable and bounded observables, is presented and illustrated for reasoning about the random results of projective measurements made on a given quantum state. Simultaneous measurements are assumed to imply that the underlying observables are compatible. A sound and weakly complete axiomatization is provided relying on the decidable first-order theory of real closed ordered fields. The proposed logic is proved to be a conservative extension of classical propositional logic.

Keywords: quantum logic, probabilistic logic, epistemic logic.

AMS MSC2010: 03G12, 81P10, 03B48.

Sorting the π^{ref}-calculus

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Communicating processes that share mutable data are everywhere. Examples range from geo-replicated databases to the internet.

In this work we develop a model for concurrent computation with references: the π^{ref}-calculus. This extends Milner's process theory. In the π^{ref}-calculus processes share mutable data through communication channels. Mobility of processes affects both the communication and the sharing topology.

We want to guarantee that processes do not manipulate undefined references, thereby compromising safety. We define a *sorting logic* (SL). In SL we judge if a process is well-sorted given the sorts of the communication channels. Well-sorted processes use their channels in a consistent way. We show that SL is sound. And we conclude by proving that processes that are judged to be well-sorted are safe.

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Diagonalization

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Self-reference and diagonalization reasonings are a very important part of Logic reasonings and appear in a great variety of contexts. For example, this kind of reasonings appears in the proof of Gödel's First Incompleteness Theorem. Self-reference is commonly attributed as the main cause of various paradoxes — Russell's Paradox, the Liar, Curry's Paradox, among others. Smullyan (2) discovered a common origin to diagonalization reasonings and Serény (1) discovered a common structure that is in the origin of the Liar.

The aims of our investigation are:

- to find a common structure for Paradoxes;
- to trace (almost) all forms of diagonalization reasonings of Mathematics to a Smullyan's style of reasoning;
- to harmonise Smullyan's reasoning with Serény's structure by means of the common structure for Paradoxes.

This work will be published in the Portuguese Journal *Revista Portuguesa de Filosofia*.

Keywords: Diagonalization, Paradox, Löb, Liar.

MSC2000: 03A10, 03B45, 03B10

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Essential structure of proofs as a measure of complexity

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The essential structure of proofs is proposed as the basis for a measure of complexity of formulas in FOL. The motivating idea was the recognition that distinct theorems can have the same derivation modulo some non essential details. Hence the difficulty in proving them is identical and so their complexity should be the same. We propose a notion of complexity of formulas capturing this property. With this purpose, we introduce the notions of schema calculus, schema derivation and description complexity of a schema formula. Based on these concepts we prove general robustness results that relate the complexity of introducing a logical constructor with the complexity of the component schema formulas as well as the complexity of a schema formula across different schema calculi.

Keywords: Description complexity, schema calculus and derivation, uniform and non-uniform robustness results.

AMS MSC2010: 03F20, 03F03, 03B10, 03B22.

Layered logics, coalgebraically

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A plethora of logics is used in Software Engineering to support the specification of systems' requirements and properties, as well as to verify whether, or to what extent, they are enforced in specific implementations. Broadly speaking, the logics of dynamical systems are *modal*, i.e. they provide operators which qualify formulas as holding in a certain *mode*. In mediaeval Scholastics such modes represented the strength of assertion (e.g. 'necessity' or 'possibility'). In temporal reasoning they can refer to a future or past instant, or a collection thereof. Similarly, one may express epistemic states (e.g. 'as everyone knows'), deontic obligations (e.g. 'when legally entitled'), or spatial states (e.g. 'in every point of a surface').

Regarding dynamical systems as transformations of state spaces according to specific transition shapes, i.e. as coalgebras for particular functors [6] such modes refer to particular configurations of successor states as defined, or induced, by the coalgebra dynamics. Coalgebra provides a *uniform* characterisation inducing 'canonical' notions of modality and the corresponding logic with respect to the underlying functor [2,3]. General questions in modal logic, such as the trade-off between expressiveness and computational tractability, or the relationship between logical equivalence and bisimilarity, can be addressed at this (appropriate) level of abstraction. In this sense, modal logic is essentially coalgebraic [1].

This talk revisits a logic suitable to express properties of, and reason about, n-layered, hierarchical transition systems, from a coalgebraic perspective, building on previous results reported in references [4,5]. In particular it is shown how the *hierachical condition*, informaly stated under the *motto* 'upper transitions should be traceable in the layer below', can be expressed as a naturality condition in the models.

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Author Index