# Corolários do T. Fubini:

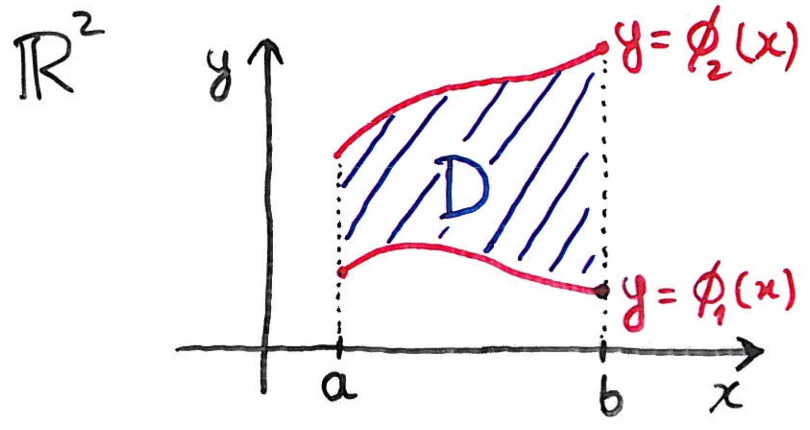
## 1 Regiões verticalmente simples

Para $f: D \longrightarrow \mathbb{R}$ contínua, com

$$D = \{(x,y) \in \mathbb{R}^2 : a \leq x \leq b,\ \phi_1(x) \leq y \leq \phi_2(x)\}$$

e $\phi_i : [a,b] \longrightarrow \mathbb{R}$ contínuas, tem-se

$$\int_D f(x,y)\, d(x,y) = \int_a^b \left( \int_{\phi_1(x)}^{\phi_2(x)} f(x,y)\, dy \right) dx$$




**Exemplo:** Calcular $\int_D xy + x^3 \, d(x,y)$, onde

$$D = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\ x^2 \leq y \leq -x+3\}$$

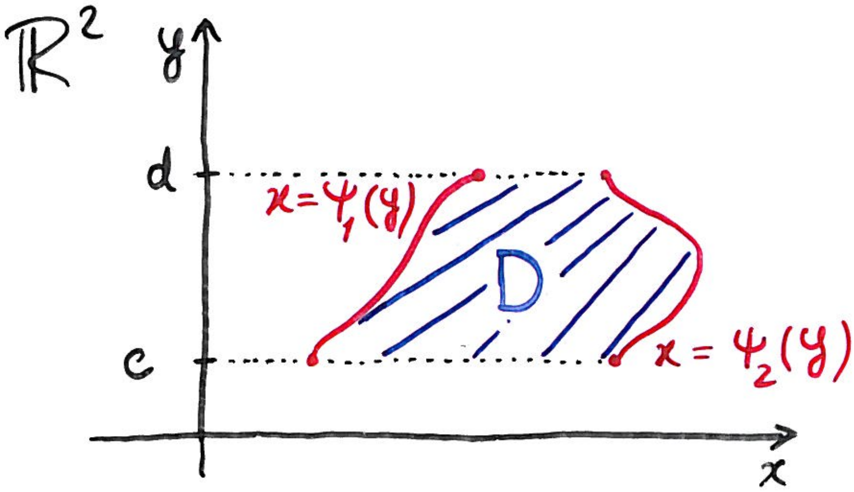
## 2 Regiões horizontalmente simples

Para $f: D \longrightarrow \mathbb{R}$ contínua, com

$$D = \{(x,y) \in \mathbb{R}^2 : \psi_1(y) \leq x \leq \psi_2(y),\ c \leq y \leq d\}$$

e $\psi_i : [c,d] \longrightarrow \mathbb{R}$ contínuas, tem-se

$$\int_D f(x,y)\, d(x,y) = \int_c^d \left( \int_{\psi_1(y)}^{\psi_2(y)} f(x,y)\, dx \right) dy$$



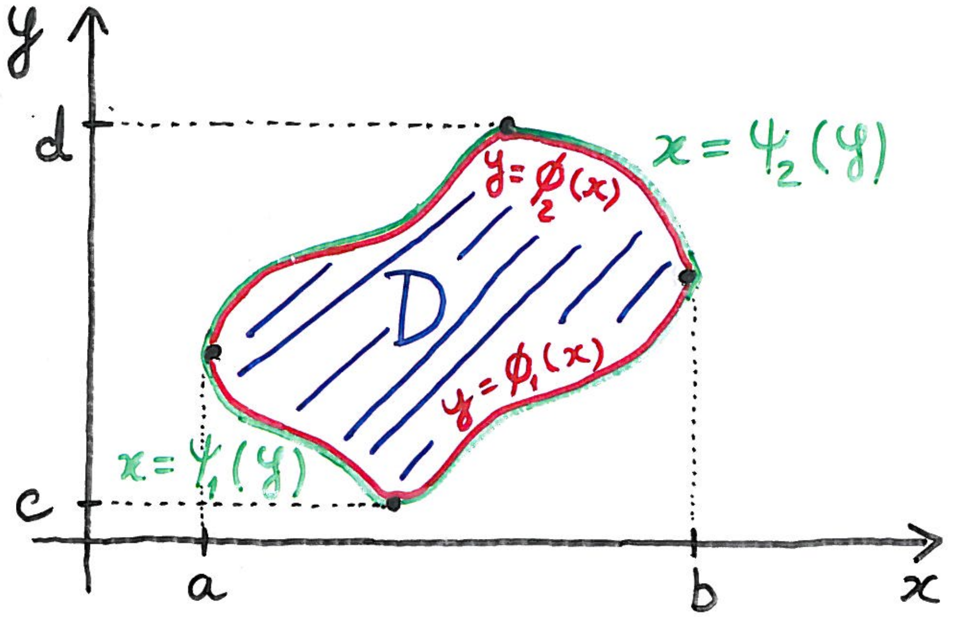

**Exemplo:** Calcular $\int_B 2(y^2+1)x \, d(x,y)$,

onde

$$B = \{(x,y) \in \mathbb{R}^2 : \sqrt{y} \leq x \leq y+2,\ 0 \leq y \leq 1\}$$

# Inversão da ordem da integração

Para $f: D \longrightarrow \mathbb{R}$ contínua, com o domínio $D$ representado de seguida




tem-se

$$\int_D f(x,y)\, d(x,y) = \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f(x,y)\, dy\, dx$$

e

$$\int_D f(x,y)\, d(x,y) = \int_c^d \int_{\psi_1(y)}^{\psi_2(y)} f(x,y)\, dx\, dy$$