Boundedness and Global Exponential Stability for Delayed Differential Equations with Applications

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We establish the boundedness of solutions for a general class of n-dimensional differential equations with distributed delays,

$$\dot{x}(t) = f(t, x_t), \quad t \geq t_0, \tag{1}$$

with $f : D \subseteq \mathbb{R} \times C_n \to \mathbb{R}^n$ continuous and $C_n := C([-\tau, 0]; \mathbb{R}^n)$, $\tau > 0$, equipped with the supremum norm, by assuming the existence of instantaneous negative feedbacks which dominate the delay effect.

For the particular situation

$$\dot{x}(t) = -\rho_i(t, x_t)[b_i(x_i(t)) + f_i(t, x_t)], \quad t \geq 0,\ i = 1, \ldots, n, \tag{2}$$

where $b_i : \mathbb{R} \to \mathbb{R}$, $\rho_i, f_i : [0, +\infty) \times C_n \to \mathbb{R}$ are continuous, some criteria for the global exponential stability of equilibria are obtained. The results are illustrated with application to delayed neural networks and population dynamics models.

We emphasize that, contrary to the usual, we do not use Lyapunov functionals to obtain our results, which can therefore be applied to a very general setting.