Global attractivity for scalar delayed differential Equations

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Abstract

For a scalar delayed differential equation in the general form $\dot{x}(t) = f(t, x_t)$, we give sufficient conditions for the global attractivity of its zero solution. It is assumed a 3/2-condition and the following generalized Yorke condition:

(YC) there exist piecewise continuous functions $\lambda_1, \lambda_2 : [0, \infty) \to [0, \infty)$ and a constant $b \geq 0$ such that, for $r(x) := \dfrac{-x}{1 + bx}$, $x > -1/b$, then

$$\lambda_1(t) r(M(\varphi)) \leq f(t, \varphi) \leq \lambda_2(t) r(-M(-\varphi)), \text{ for } t \geq 0,$$

where the first inequality holds for all $\varphi \in C := C([-\tau, 0]; \mathbb{R})$ and the second one for $\varphi \in C$ such that $\varphi > -1/b \in [-\infty, 0)$, and $M(\varphi) := \max\{0, \sup_{\theta \in [-\tau, 0]} \varphi(\theta)\}$ is the Yorke's functional.

The hypotheses imposed are weaker than the ones in the recent works [1] and [2]. The results are applied to obtain several criteria for the global attractivity of the positive equilibrium for some well-known "food-limited" population models with delay.
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