The multiary version of the λ-calculus with generalised application

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Extended Abstract

In [1] we defined the generalised multiary λ-calculus $\lambda\mathbf{J}^{\mathbf{m}}$ with the purpose of extending Schwichtenberg's work on permutative conversions for intuitionistic cut-free sequent calculus [4]. $\lambda\mathbf{J}^{\mathbf{m}}$ comes equipped with a set of permutative conversions for which the permutability theorem holds: two $\lambda\mathbf{J}^{\mathbf{m}}$-terms determine the same λ-term iff they are inter-permutable. We established confluence and strong normalisation of these conversions.

The generalised multiary λ-calculus $\lambda\mathbf{J}^{\mathbf{m}}$ is an extension of the λ-calculus where application is generalised in two directions: (i) "generality", in the sense of von Plato's generalised eliminations [5]; and (ii) "multiarity", *i.e.* the ability of applying functions to lists of arguments. Thus the λ-calculus with generalised application ΛJ of Joachimski and Matthes [3] may be seen as a notational variant of a subsystem of $\lambda\mathbf{J}^{\mathbf{m}}$ called $\lambda\mathbf{J}$.

Generality may express multiarity and this fact detarmines a mapping ν from $\lambda\mathbf{J}^{\mathbf{m}}$ to $\lambda\mathbf{J}$. Also, multiarity can express certain uses of generality, determinig a mapping μ which calculates the normal forms for the reduction rule of $\lambda\mathbf{J}^{\mathbf{m}}$ with the same name. It follows that μ and ν are inverse bijections between μ-normal forms and terms of $\lambda\mathbf{J}$. We develop this idea and investigate how these mappings preserve reduction. This emphasis on how multiarity and generality may express each other contrasts with that in [1], where multiarity and generality are studied as independent features of $\lambda\mathbf{J}^{\mathbf{m}}$.

Our investigations of the relationship between generality and multiarity identify two isomorphic subsystems of $\lambda\mathbf{J}^{\mathbf{m}}$: (i) a variant of $\lambda\mathbf{J}$, which is the subsystem with minimal use of multiarity (*i.e.* no use); (ii) the subsystem of μ-normal forms, which is the subsystem with maximal use of multiarity (*i.e.* uses multiarity for expressing generality whenever possible). The two isomorphic subsystems are non-redundant opposite extremes w.r.t. the use of multiarity.

However both subsystems have shortcomings because of this extreme nature. In the former, multiarity is not available as a shorthand. In the latter, it is a simple definition of expressions and reduction that is not available, because unconstrained generalised multiary application, as well as reduction steps, can create μ-redexes, *i.e.* do not preserve maximal multiarity. Although exhibiting some redundancy, $\lambda\mathbf{J^m}$ does not suffer from the drawbacks of these subsystems. Therefore it seems to be the system with the right use of multiarity.

As an application of our study of mappings ν and μ, we lift to $\lambda\mathbf{J^m}$ Joachimski and Matthes results [2, 3] of confluence and strong normalisation for ΛJ.

References

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