An isomorphism between sequent calculus and natural deduction for minimal logic

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Abstract

Roughly speaking, the traditional mappings between sequent calculus and natural deduction translate an elimination as a combination of a left introduction and a cut; translate a left introduction as a combination of an elimination and substitution; and translate cut as substitution. We show, for minimal logic, how to formulate sequent calculus and natural deduction in such a way that both systems have a cut rule and a substitution operation; and the left introduction rule, the cut rule and the substitution operation of the sequent calculus "correspond" to the elimination rule, the cut rule and the substitution operation of the natural deduction system. The correspondence between rules induces a correspondence between derivations: giving a derivation in one system, and replacing each occurrence of one rule by one occurrence of the corresponding rule, one obtains a derivation in the other system (up to "turning branches upside down"). This process defines a bijection between the two sets of derivations. Moreover, we may define, with the help of substitution, simple cut elimination rules in both systems in such a way that "cut-elimination" in the natural deduction system subsumes normalisation; and the bijection between the sets of derivations becomes an isomorphism between the two cut-elimination relations. In addition: (1) The isomorphism between the two systems, when the systems are presented as extensions of the simply typed lambda-calculus, may be described as an inversion of the associativity of application. (2) Traditional natural deduction, even in the extended sense of von Plato, is a common core of our systems.