Some applications of Buchholz's Omega rule*

(Abstract)

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The goal of the course will be to present some concepts and methods of proof theory, centred around various applications of the so-called Ω-rule.

The negative translation due to Kolmogorov 1925, and rediscovered by Godel and Gentzen 1932, shows that one can reduce the classical theory of first-order arithmetic to its intuitionistic version. One can argue that this is a positive solution to Hilbert's program (and it is actually interesting to understand the connection with Godel second incompletness theorem). Similar translations work for other systems, for instance, second-order arithmetic. However, there are cases where such a translation cannot be applied. In some cases, this is because the classical version is strictly more powerful from a proof-theoretic point of view, and hence one cannot have a syntactical translation in the intuitionistic version. But in other cases, the negative translation does not work *and* it can be shown that the classical and intuitionistic versions have the same proof-theoretical power. One example is arithmetic restricted to Σ^0_1 induction. Another example is the so-called theory of finitely iterated inductive definitions. Buchholz found a general technique, called the Ω-rule, to show in this case that the strength of the classical version is the same as the intuitionistic version. This method can be presented as a refinement of the negative translation, and can be applied to other frameworks, for instance Σ^0_1 induction. A similar method applies also to reduce some fragment of second-order arithmetic to intuitionistic theory of inductive definitions. This can be presented in a suggestive way as an analysis of some fragment of system F and we describe in particular a natural fragment of system F which has the same strength as first-order arithmetic.

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