# Applicative Theories

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In the 1970s Solomon Feferman introduced *Explicit Mathematics* in order to give a logical account to Bishop-style constructive mathematics. This framework plays an import role in the proof-theoretic analysis of subsystems of second order arithmetic and set theory. It is formulated in a two-sorted logic which permits to introduce theories with strong mathematical expressive power by use of set existence axioms.

Here we focus on the first order part of explicit mathematics, called *applicative theories*, which are of interest in their own. Applicative theories are based on type-free, partial combinatory logic which permits the definition of $\lambda$ abstraction and recursion in a partial setting. They provide a uniform approach to a large class of theories of mathematical interest by change of the induction principles and by adding different types of functionals. Imposing certain restrictions it is also possible to characterize theories of bounded complexity.

Apart from these proof-theoretic features, applicative theories can be used as a logic for functional programming. In particular, the partiality of the underlying logic permits to speak directly about non-termination. With it we can, for instance, define a theory with a least fixed point operator (with respect to the definedness order) for recursive functions.

In this talk we give a survey on applicative theories, including the proof-theoretic results and some applications for functional programming.