GILDA FERREIRA, *Weak Theories of Arithmetic and the Counting Hierarchy.*
CMAF-Universidade de Lisboa.
E-Mail: gildafer@cii.fc.ul.pt

Bounded arithmetic and its connections to computational complexity theory have seen important developments in the last several years.

Theories of bounded arithmetic are formal systems where formulas and induction are strongly restricted so that provability in these theories is tightly connected to complexity classes. We can say, to a certain extent, that [1] was the work that started the study of the computational strength of formal theories of bounded arithmetic.

We are particularly interested in the class FCH, the *Hierarchy of Counting Functions*. So, after presenting a machine independent characterization of this class, inspired in [2] we introduce a second order theory in binary notation, TCA, whose provably total functions (having appropriate graphs) are exactly the functions in FCH. The proof of this relation between TCA and FCH is based in the free cut elimination theorem.

Then we introduce successively stronger theories enriching TCA with schemes like bounded collection, recursive comprehension and strict-Π^1_1 reflection, and expressing it in a more powerful second order language which enables us to consider infinite sets. In this way we obtain a weak theory for analysis that we call $TCA^2 + s\Pi^1_1$-Ref. The class of provably total functions in these successively enlarged systems is still FCH. We get this computational strength by proving some conservation results over TCA, using (in the case of the stronger theory) some forcing techniques.

Our interest in TCA^2 and related theories is based in the conviction that TCA^2 is just enough to develop the Riemann integral.

[1] Buss S., *Bounded Arithmetic*, Bibliopolis (1986).

[2] Johannsen J., Pollett C., “On Proofs About Threshold Circuits and Counting Hierarchies (Extended Abstract)”, *Thirteenth Annual IEEE Symposium on LICS* (1998), pp.444-452.

[Partially supported by CMAF, POCTI, FCT and FEDER]