

A CLASS OF RIEMANNIAN INTEGRABLE CONTINUOUS FUNCTIONS IN WEAK ANALYSIS

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ABSTRACT. In the paper "Groundwork for weak analysis" Fernandes and Ferreira develop some weak analysis in the sense that they formalize and develop analysis in a weak - that is sub-exponential, mainly polynomial time computable - theory. They develop the real number system proving it is a closed ordered field and define continuous function. In that paper, the authors raise the question of finding out the amount of analysis that can be performed in this weak systems of second order arithmetic.

As the authors pointed out Riemannian integration can't be fully performed for continuous functions in those weak theories, but there should be a significant class of continuous functions for which we can perform such operation. Walking in this direction we define a class of continuous functions, which is a subclass of that defined by Fernandes and Ferreira, using polynomial approximation and for which we perform Riemannian integration.

We identify some functions belonging to this class such as the trigonometric sine and cosine and we show that the class is significant in the sense it is closed to sum, product or composition. We also prove the fundamental theorem of the calculus for the above mentioned class of continuous functions.

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