The Curry/Howard correspondence and some of its extensions to sequent calculus*

(Abstract)

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The Curry/Howard correspondence, in its simplest form, establishes a connection between natural deduction for implicational intuitionistic logic and the simply typed λ-calculus. In this course, we present some experiments in the attempt to obtain an analogous correspondence between sequent calculi and appropriate extensions of the simply typed λ-calculus.

We review natural deduction and sequent calculus for intuitionistic logic, with emphasis on two issues: (i) permutability of inferences in sequent calculus and the permutability theorem establishing that two sequent calculus derivations may be interpreted as the same natural deduction proof iff they are inter-permutable; (ii) the relationship between normalisation and cut-elimination. We make precise the Curry/Howard correspondence, showing how to read the typing system and β-reduction of the simply typed λ-calculus as a natural deduction system (with term annotations to represent proofs) and its normalisation procedure. We survey some of the attempts to extend the Curry/Howard correspondence to sequent calculus, with special focus on Herbelin's work. We argue that these approaches do not address directly the permutability phenomenon in sequent calculus.

We present a system, the generalised multiary λ-calculus, which is an extension of the simply typed λ-calculus where application is generalised in two directions: (i) "generality", in the sense of von Plato's generalised eliminations; and (ii) "multiarity", i.e. the ability to apply functions to lists of arguments. We show how to view this system as a sequent calculus with term annotations, where the rule for typing the new form of application, when viewed as an inference rule, encompasses both a form of left introduction and a class of cuts. Our approach has an account of the permutability phenomenon in that the generalised multiary λ-calculus comes equipped with a confluent and strongly normalising set of permutations, for which the permutability theorem holds.

*Course to be lectured at the meeting *Days in Logic*, 22-24 January 2004, Braga, Portugal