The Einstein equation in spacetimes with density

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We consider a Lorentzian manifold (M, g) equipped with a density function f. This density modifies the Riemannian volume giving rise to a smooth metric measure space $(M, g, e^{-f} dvol_g, \mu)$, where μ is an extra parameter. From a geometric viewpoint, new curvature tensors that encode information about the density shall be defined. Thus, taking as a starting point the Bakry-Émery Ricci tensor $\rho^f = \rho + \mathrm{Hes}_f - \mu df \otimes df$, *weighted* analogues of the usual curvature tensors (Schouten tensor, scalar curvature,. . .) can be found in the literature. The choice of this tensors is done so that they retain some properties of their Riemannian counterparts. In this context, a natural question is what would be a reasonable generalization of the Einstein tensor. The first purpose of this talk is to give an answer to this question.

As a second aim, we study the geometry of solutions to the resulting vacuum weighted Einstein field equations. Two cases are distinguished: the isotropic (i.e. ∇f is a null vector field) and the non-isotropic (i.e. ∇f is spacelike or timelike). We analyze geometric properties of the solutions and show that, under some assumptions, they belong to well known families of spacetimes.