On the topological complexity of certain connected sums

The study of the topological complexity (TC) of the Klein bottle and of connected sums of projective spaces shows evidence that the non-commutativity of their fundamental group is related to the maximality of their TC. On the other hand, it has been proven that for certain abelian groups G, any manifold $\mathcal{M}$ with fundamental group G has non-maximal TC. Motivated by the study of $\mathbb{T}^3\#\mathbb{T}^3$, and more generally of connected sums of "classic" manifolds in dimension 3, I will show techniques and developments concerning the maximality of TC for certain connected sums, in relation with specific free products of fundamental groups.