DAYS IN λOGIC 2026

29 - 31 January | Universidade do Minho | Braga | Portugal

Booklet of Abstracts

Courses and Contributed Talks

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Dynamic Epistemic Logic for Formal Epistemology

Cyclic proofs, a primer

Anupam Das
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'Cyclic' and 'non-wellfounded' proofs have emerged in the last 30 years as a powerful mechanism to reason about systems with (co)induction. They are an alternative to usual well-founded proofs with (co)induction principles, that are seemingly more powerful at the level of expressivity, and more intuitive at the level of proof search, often avoiding elusive invariants. In this course we shall introduce students to the foundations of cyclic proof systems. We shall present the core theory, based on seminal results in the literature. Moreover we shall give the students the tools they need to apply these techniques to new logics and theories.

Higher-order modal logic and the philosophy of mathematics

Bruno Jacinto
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Modality has been influential in addressing a number of perennial philosophical questions, and there has recently been a resurgence of philosophical interest in higher-order logic, especially in so-called higher-order metaphysics. In this tutorial we present and address important technical-cum-philosophical questions concerning the interaction of modality with higher-order quantification. Moreover, we show how the appeal to higher-order modal resources paves the way to Upper Logicism, a novel form of logicism based on the Russellian view that mathematical entities are logically definable entities at higher-orders.

Dynamic Epistemic Logic for Formal Epistemology

Sonja Smets
University of Amsterdam, The Netherlands

In this tutorial, we explore how the tools of logic can shed new light on central epistemological questions. We focus on formal approaches to qualitative belief representation, belief revision, knowledge updates, interactive learning, and doxastic group attitudes. By bridging our investigation with research in Dynamic Epistemic Logic, Belief Revision Theory and Formal Learning Theory, we uncover essential connections across these domains. Merging insights from these areas also allows us to address questions in social epistemology by focusing on the social group dynamics. Beyond philosophy, these frameworks have direct relevance for the logical study of multi-agent systems in AI.

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Contributed Talks

Ana Jorge Almeida (LIACC and Faculdade de Ciências da Univ. Porto, Portugal), with Mário Florido and Sandra Alves
On the relation between a calculus with explicit substitutions and a resource calculus

Carlos Caleiro (SQIG – Instituto de Telecomunicações, Departamento de Matemática, Instituto Superior Técnico, and Univ. Lisboa, Portugal), with Sérgio Marcelino
Bounded rationality and polynomial approximations beyond classical logic

Cheng-Syuan Wan (Tallinn University of Technology, Estonia)
An Agda formalization of nonassociative Lambek calculus

Daniel Graça (Univ. Algarve and Center for Research and Development in Mathematics and Applications, Portugal)
Analytic maps and Turing universality

Filipa Mendes (Centre of Mathematics, Univ. Minho, Portugal), with José Espírito Santo
The logical essence of call-by-name CPS translations

Imme van den Berg (Univ. Évora, Portugal)
A nonstandard functional approach to the measure decomposition theorem

Jin Wei (Technische Universität Darmstadt, Germany)
Approximate Completeness of Hypersequent Calculus for First-Order Łukasiewicz Logic

Juliana Cunha (CIDMA, Dep. Mathematics, Univ. Aveiro, Portugal), with Alexandre Madeira and Luís S. Barbosa
Modal invariant relations for paraconsistent transition systems

Miguel Alves (Departamento de Matemática, Univ. Minho, Portugal)
On the mechanisation of the multiary λ-calculus and subsystems

Muhammad Afaq Khan (CIDMA & Department of Mathematics, Univ. Aveiro, Portugal), with Manuel Martins and Alexandre Madeira
On fuzzy topological semantics

Pedro Pinto (TU Darmstadt, Germany)
Moduli and quantitative aspects of hyperbolic spaces via proof mining

Rodrigo Alves (CIDMA, Univ. Aveiro, Portugal), with Alexandre Madeira and Manuel Martins
The category of many-logics modal logic models

Sérgio Marcelino (SQIG – Instituto de Telecomunicações, Departamento de Matemática, Instituto Superior Técnico, and Univ. Lisboa, Portugal)
Can we effectively compare many-valued logics?

On the relation between a calculus with explicit substitutions and a resource calculus

Ana Jorge Almeida, Mário Florido
LIACC and Faculdade de Ciências da Universidade do Porto
Sandra Alves
CRACS, INESC-TEC and Faculdade de Ciências da Universidade do Porto

Term expansion was originally introduced in 2004 [4] as a way to relate terms typed in an intersection type system with linear terms. Recently, new applications of term expansion include the relation of lambda-terms with terms typed in other substructural type systems [1] (relevant and ordered type systems) and the use of quantitative types to relate the strongly normalising lambda-terms with weak linear terms that share the same normal form [2].

Here we define a new term expansion for a calculus with explicit substitutions [5], using it to relate a lambda-calculus with explicit substitutions to Boudol's lambda-calculus with multiplicities [3], where function arguments have a possibly limited availability. Let us look at the term $t \equiv (xx)<x := I>$ of the λ-calculus with explicit substitutions, with type $\alpha \to \alpha$, where I is the identity function. Its *expansion* is the term $(xx^1)\langle I^2/x\rangle$ in Boudol's lambda-calculus with multiplicities [3], meaning that the identity function I may be copied twice during reduction, in this case, as much as there are free occurrences of x in xx. Note that the expanded term also has type $\alpha \to \alpha$. The *expansion* of a term can thus be interpreted as a different version of the original term, where multiplicities are explicitly included in the term syntax, allowing us to meticulously track the resource aware behaviour of the term, by showing us how resources are being used throughout term evaluation.

We show that we can relate associative, commutative and idempotent intersection types with a subset of Boudol's calculus that uses only infinite multiplicities, and associative, commutative and non-idempotent intersection types with a subset that deals with finite multiplicities. These proofs demonstrate that there is an operational equivalence between standard lambda-terms typed with intersection types and resource-calculi.

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Bounded rationality and polynomial approximations beyond classical logic *

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Bounded rationality [1] is the idea that decision making is often sub-optimal due to reasoning limitations. Namely, the *cost of reasoning* as a measure of the effort required by non-ideal, resource-bounded (human or artificial) agents in order to perform non-trivial inferences, is a crucial issue in philosophy, AI, economics and the cognitive sciences.

In a series of papers [2, 3, 4, 5], d'Agostino and coauthors have tackled this problem and proposed a *depth-bounded* approach to classical (Boolean) reasoning agents, recently extended to FDE (Anderson, Belnap). Further, they have formulated the following research question.

Can we define, in a natural way, a hierarchy of logical systems that indefinitely approximate a given idealized Logic (e.g., Boolean Logic) in such a way that, in all practical contexts, suitable approximations can be taken to model the inferential power of actual, resource-bounded agents?

Herein, we overview d'Agostino et al's approach, and show that *depth-bounding* can be generalized to a myriad of non-classical logics. For the purpose, we use the results in [6, 7] to provide polynomial time approximations for any logic defined by a monadic finite-valued partial non-deterministic logical matrix, or simply by a suitable analytical calculus.

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An Agda formalization of nonassociative Lambek calculus

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In this talk, we present a formalization of nonassociative Lambek calculus [2] in the Agda proof assistant. The sequent calculus for this logic has sequents with binary trees as antecedents, in which formulae are stored as leaves.

The structure of the antecedents creates subtleties when proving logical properties since in many cases one needs to analyze equalities of different presentations of the same tree. We formally characterize all the cases of how two presentations of a tree can be equal in Agda. The resulting technical lemma helps us to exhaust all possible cases when proving several meta properties of the calculus, including cut admissibility and the Maehara interpolation property [3], which implies Craig interpolation [1].

Moreover, both the cut rule and the interpolation procedure are well-defined wrt. an equivalence relation of derivations. We additionally prove the proof-relevant versions of (i) soundness and completeness with respect to the Hilbert-style (axiomatic) calculus and (ii) Maehara interpolation, i.e. the interpolation procedure as a right inverse of the admissible cut rule.

This talk is based on joint work with Niccolò Veltri [4].

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Analytic maps and Turing universality

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The iteration of quadratic maps such as the logistic map in the unit interval is well-known to originate complex dynamical behaviour, namely chaotic behaviour. In the 1990s Moore asked whether the iteration of analytic maps would be complex enough to not only generate chaotic behaviour, but also to be Turing universal on compact sets, and conjectured that the answer was negative [1].

In a remarkable result, it was shown that the simulation of Turing machines on the compact set $\mathbb{S}^{17}$ is possible if a slowdown of the simulation is allowed and if the encoding of configurations is not point-to-point.

In this talk we explore this question and related results and show that, under fairly general conditions, the answer to Moore's conjecture is negative on $[0,1]^n$ for every $n \in \mathbb{N}$ if no slowdown on the simulation is allowed and if the encoding of configurations is done point-to-point.

On a positive note, we show that Turing machines working with bounded space can be simulated by an analytic map with no slowdown on $[0,1]^2$ and that this simulation is robust to perturbations.

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The logical essence of call-by-name CPS translations

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Among the many manifestations of the Curry-Howard isomorphism [1], there is the link between double-negation (DN) translations and continuation-passing style (CPS) translations [2]. DN translations are interpretations of classical logic into intuitionistic logic [1, 3]. CPS is a compilation technique for high-level programming languages [4]. The "essence" of the CPS translation is a conversion of the source program to so-called administrative normal form [5, 6]. The "logical essence" of the CPS translation is a conversion of the source program, from the natural deduction format to the sequent calculus format, hence a change of deductive system without change of logic [7, 8]. In this view, the DN translation is an optional, later encoding of the sequent calculus into the lambda-calculus; and composing the format conversion with the optional encoding gives back the CPS translation. These results give a theoretical basis to the proposal of the sequent calculus as an intermediate language for compilation [9]. We will restrict the presentation to the case when the source program follows the call-by-name evaluation strategy.

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A nonstandard functional approach to the measure decomposition theorem

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A classical decomposition theorem of measure theory states that a finite measure on an interval $[a, b]$, say, is the sum of an atomic, singular and regular measure. The regular part may be represented by a density function, but the atomic and singular part cannot be functionally represented.

We present alternative decomposition theorems, in the setting of an infinitely fine discretization $[a..b]$ of the interval, with steps $\delta y \simeq 0$. A function $f : [a..b[\to \mathbb{R}^+$ is *of limited accumulation* if $\sum_u^v f(y)\delta y$ is limited, whenever $u \simeq v$. Due to a property of logic (Internal Set Theory IST does not contain external Δ_1-formulas), the discrete integral $I \equiv \sum_a^b f(y)\delta y$ is still limited. It has a representation

$$I \simeq A + S + R,$$

with A an atomic contribution, S a singular contribution and R a regular contribution. All parts are represented by density functions, a simple example of an atomic function being the Dirac-like function Δ_0, taking its only non-zero, infinitely large value $1/\delta y$ at zero. A simple example of a singular function uses points at distance $\sqrt{\delta y}$, where it takes its only non-zero, infinitely large values $1/\sqrt{\delta y}$. Decompositions of the interval can also be given, in terms of supports.

In particular, we discuss which seems to be an enlightening decomposition into external sets: $[a..b[= \gamma \cup \sigma \cup \rho$, where

$$\begin{aligned}
\gamma &= \left\{ u \in [a..b[: f(u) \text{ unlimited}, (\exists z \simeq 0) \left(\sum_{u-z}^{u+z} f(y)\,\delta y \gtrsim\!\!\!\!\not\;\; 0 \right) \right\} \\
\sigma &= \left\{ u \in [a..b[: f(u) \text{ unlimited}, (\forall z \simeq 0) \left(\sum_{u-z}^{u+z} f(y)\,\delta y \simeq 0 \right) \right\} \\
\rho &= \{ u \in [a..b[: f(u) \text{ limited} \}
\end{aligned}$$

On γ, the atomic contribution A may be approximated by a discrete integral up to an arbitrarily small standard number ε, both from the inside and the outside, and similar properties hold for the singular contribution S on σ, and the regular contribution R on ρ.

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Approximate Completeness of Hypersequent Calculus for First-Order Łukasiewicz Logic

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Introduced by Jan Łukasiewicz in the 1930s, (infinite-valued) Łukasiewicz logic, one of the fuzzy logics where formulas take values in the closed interval $[0, 1]$, has been extensively studied and in recent years has found significant applications in continuous model theory. Although Łukasiewicz logic and its first-order variant admit a simple Hilbert-style axiomatization, their proof-theoretic analysis faces substantial difficulties. A key obstacle is the failure of the deduction theorem, arising from incompatibility with the contraction principle.

Gentzen-style proof systems for Łukasiewicz logic were first developed by by Metcalfe, Olivetti, and Gabbay in [1] in the form of hypersequent calculus and was later extended to first-order setting by Baaz and Metcalfe in [2]. As later observed and corrected by Gerasimov in [3], the proof of first-order (approximate) completeness appeared in [2] applies only to prenex formulas, leaving open the question of completeness for arbitrary first-order formulas.

In this talk, I will present recent results from my work in [4], in which I generalize the approach in [1], most notably an extension of the appropriate Herbrand's theorem, to establish full first-order completeness and consequentially an approximate form of cut admissibility for the hypersequent calculus. In fact, the result was stronger in the sense that it yields completeness with respect not only to first-order validity but also to provability in the form of arbitrary hypersequents. Finally, I will outline several directions for future research, including the development of a cut elimination procedure and a constructive fragment of Łukasiewicz logic, with potential applications to proof mining analysis of ultrapowers in continuous logic.

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Modal Invariant Relations for Paraconsistent Transition Systems

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Unlike classical logic, paraconsistent logics allow reasoning in the presence of inconsistency without collapsing into triviality. They are particularly suitable for modelling scenarios where information comes from multiple sources or is affected by external factors, and may therefore be incomplete or contradictory. To address such situations, we propose Paraconsistent Transition Systems, parametric over a class of residuated lattices [1]. In these systems, transitions are weighted by two values representing the evidence for and against their traversal. Since these values are not necessarily complementary, it becomes possible to represent different epistemic scenarios: vagueness, consistency, and contradiction.

In this talk, we outline the motivation behind this framework and present the associated paraconsistent modal logic. Additionally, since bisimulation is a key notion for studying behavioural equivalence in transition systems, we introduce two notions—crisp and graded bisimulation—to relate Paraconsistent Transition Systems [2]. Finally, we discuss modal invariance results for specific subsets of formulas.

(Joint work with Alexandre Madeira and Luís S. Barbosa)

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On the mechanisation of the multiary λ-calculus and subsystems

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What happens when we consider a variant of the λ-calculus where applicative terms have lists of arguments instead of single arguments? Surprisingly, this simple idea has a logical correspondence through the lens of the Curry-Howard isomorphism, and is offered by the multiary λ-calculus system, named λm [4]. This multiary λ-calculus can be seen as a restricted version of ΛJ, the λ-calculus with generalised applications [2, 3]. A special reduction rule of λm is a particular case of ΛJ's commuting conversion rule [2, 6]. Moreover, one can also notice the similarities between the systems λm and $\overline{\lambda}$ [1], where the latter is a calculus of proof terms for the focused sequent calculus *LJT*.

In this talk we will give a brief overview of the multiary λ-calculus λm and pay particular attention to the problem of finding a subsystem isomorphic to the (simply-typed) λ-calculus. There is one such subsystem, a kind of vectorial notation where substitution is necessarily unconventional. Next, we will present a mechanisation of these systems and their metatheory using the *Rocq Prover* [7] and the *Autosubst* library [5]. In particular, we will motivate our main mechanisation choices (which include the use of de Bruijn indices to deal with binders and parallel substitutions) and highlight some differences when comparing to the original pen-and-paper definitions and proofs. (This talk is based on my MSc Thesis, supervised by José Espírito Santo and Luís Pinto.)

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On Fuzzy Topological Semantics

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Topological semantics [1] interprets modal operators using open sets in a space, but in its classical form it only allows crisp truth values and sharp boundaries. A formula is either true or false at a point, and regions where it holds are ordinary open sets. This makes it difficult to model vague, graded or uncertain information that appears in many real systems. In contrast, many valued and fuzzy modal logics already work with degrees of truth, but most of their semantics are based on fuzzy Kripke frames, which focus on accessibility relations rather than spatial structure [2, 3, 4]. In this talk, we present a fuzzy topological semantics for many valued modal logic based on Gödel algebra. In this setting, each formula is interpreted as a fuzzy subset of a fuzzy topological space, so that every point in the space is assigned a degree of truth between 0 and 1. Modal operators are evaluated using fuzzy interior and fuzzy closure operations, while the propositional connectives follow the standard Gödel operations. This preserves the spatial intuition of classical topological semantics (reasoning in terms of "regions" and "neighbourhoods"), in this context the regions can be graded. A formula can be more true at some points and less true at nearby points. This makes the semantics suitable for modelling imprecise observations, partial knowledge and approximate locations. On the semantic side, we show how reflexive and transitive fuzzy Kripke frames give rise to fuzzy Alexandroff spaces, which provides a direct bridge between fuzzy relational and fuzzy topological models. Within this framework, we prove that the modal system $S4$ is sound for the class of all fuzzy topological spaces [5]. The resulting approach offers a flexible and unified tool for reasoning about graded modalities, with potential applications in artificial intelligence, epistemic reasoning and spatial logic, where both uncertainty and underlying spatial structure play an important role.
Acknowledgment: This is a joint work with Manuel Martins and Alexandre Madeira.

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Moduli and Quantitative Aspects of Hyperbolic Spaces via Proof Mining

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Proof mining is a research program that traces its origins back to Kreisel's work in the 1950s and was later developed in a systematic and influential way by Kohlenbach. Its central aim is to apply techniques from Proof Theory to the analysis of mathematical proofs, not merely to verify their correctness, but to extract hidden computational information that is often implicit in nonconstructive arguments. In this way, proof mining sheds new light on classical proofs and frequently leads to strengthened results, such as explicit bounds, rates of convergence, or effective uniformities that were previously inaccessible.

In this talk, we will discuss how a proof-theoretical perspective has led to the introduction and refinement of several key notions in functional analysis. Remarkably, these notions have turned out to be not only technically convenient but also conceptually natural and highly suitable for ongoing research in analysis and related areas.

The main focus of the talk will be on certain classes of hyperbolic spaces and on associated quantitative parameters, such as moduli encoding geometric or convexity properties. We will explain how these moduli play a central role in proof mining applications and how they facilitate the extraction of uniform quantitative information. Recent results will be presented to illustrate the effectiveness of this approach and to demonstrate how the properties discussed lead to new insights and applications beyond their original logical motivation.

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The Category of Many-Logics Modal Logic Models*

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Many-Logics Modal Logic (MLML) provides a framework for reasoning about complex systems where states can be governed by different logics [2]. This heterogeneity is managed by associating each state with a specific sublattice of a common global lattice L [1]. While recent work has formalised MLML as an L-institution [3], the standard definition of the category of models $Mod^{MLML}(\Sigma)$ requires a strict condition for model morphisms: the lattices associated with related states must be identical, typically expressed as $I(w) = I'(h(w))$.

In this talk, we propose a less restrictive version of the category of models. Specifically, we propose to map models using a function that maps states to states and a family of lattice homomorphisms indexed by the states, allowing the lattice of one state to be mapped to the lattice of the corresponding state in the other model. By relaxing the morphism requirements in this way, we allow for a more flexible relationship between local configurations, a need motivated by the transition between models with different levels of "classicality" or uncertainty.

As part of this investigation, we explore the products within this category. We consider the construction of products of a model with itself, as well as products between different models, examining the structural conditions necessary for these categorical operations.

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Can we effectively compare many-valued logics?

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Partial non-deterministic logical matrices (PNmatrices), where connectives are interpreted as multi-functions, greatly extend the expressive power of many-valued semantics and support modular, algorithmic constructions of non-classical logics. A basic question is then: given two finite PNmatrices, can we effectively compare the logics they determine?

For ordinary (deterministic) finite matrices the answer is positive. However, as soon as genuine non-determinism is allowed, the picture changes drastically. When logic is understood as a set of theorems (Fmla-logics) or as a single-conclusion Tarskian consequence relation (Set $\times$ Fmla-logics), the corresponding comparison problems are undecidable [1, 2]. When considering logics as multiple-conclusion consequence relations (Set $\times$ Set-logics), the computational status of the comparison problem remains an open question.

We can, however, obtain positive results by restricting the language to a fixed finite number of variables. Recent work [3] shows that while the n-variable comparison problems for Fmla- and Set $\times$ Fmla-logics remain undecidable, the problem becomes decidable for Set $\times$ Set-logics. The proof reduces the comparison of n-variable Set $\times$ Set-fragments to the validity of formulas in the Weak Second-order Logic of k Successors (WSkS), which is known to be decidable [4]. This provides an effective method for comparing Set $\times$ Set-logics of finite PNmatrices whenever they admit (multiple-conclusion) axiomatizations using a finite, explicitly bounded number of variables. Do such axiomatization always exist?

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