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Near Linear Algebra

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Abstract

In quasi-linear algebra, we study objects that generalize vector spaces in the sense that one of the distributive laws fails, yet a "good" notion of basis is still available. In this talk, we will explain which classical linear-algebraic results continue to hold and what the main differences are. We will see that quasi-linear algebra opens the door to genuinely non-linear phenomena while preserving most of the advantages of linear algebra, offering a potentially more flexible working framework.

The ω-word problem over the pseudovariety of aperiodic block groups

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Abstract

An ω-term is a formal expression obtained from the letters of an alphabet using the operations of multiplication and ω-power. The ω-word problem for a pseudovariety V of finite semigroups consists in deciding whether two ω-terms have the same interpretation over every semigroup of V.

The ω-word problem has been solved for several aperiodic pseudovarieties. The case of the pseudovariety J of all J-trivial semigroups, solved by J. Almeida [1], constitutes a classical example. Another notable example is that of the pseudovariety A of all aperiodic semigroups, solved by J. McCammond [3] by transforming each ω-term into a certain normal form and showing that different normal forms have different interpretations in some finite aperiodic semigroup. The decidability of the word problem for ω-terms was also obtained for the pseudovariety $A \cap ECom$, of aperiodic semigroups whose idempotents commute, by the speaker in collaboration with M. Branco [2]. The characterization of the ω-terms over $A \cap ECom$ is given by certain reduced automata associated with the terms.

Recall that a block group is a semigroup whose elements have at most one inverse. A block group can also be defined as a semigroup in which each R-class and each L-class have at most one idempotent. This talk deals with the ω-word problem over the pseudovariety $A \cap BG$, of all aperiodic block groups. This problem is reduced to considering only ω-terms of ranks 1 and 2. For rank 1, the solution over $A \cap BG$ coincides with the solution over A. For rank 2 the solution over $A \cap BG$ combines the solutions over A, J and $A \cap ECom$.

(This is an ongoing joint joint work with Conceição Nogueira and M. Lurdes Teixeira.)

References

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Matrices with hyperbolic J-numerical range

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Abstract

We consider an indefinite inner product space induced by a non-singular signature matrix J. A criterium for no degenerate hyperbolicity of the J-numerical range is presented. This shape characterizes the 2-by-2 case and persists for certain classes of structured matrices independently of their size. Results in this vein are obtained and illustrated, some of them are counterparts of classical numerical range results. This talk is based on a joint work with N. Bebiano and G. Soares.

On the mechanisation of the multiary lambda calculus and subsystems

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Abstract

In 1995, Hugo Herbelin introduced a multiary variant of the λ-calculus (named $\bar{\lambda}$), whose λ-terms correspond to proofs in a sequent calculus presentation of intuitionistic logic (through the lens of the Curry-Howard isomorphism). A simpler variant of the λ-calculus still preserving the idea of applicative terms having lists of arguments is offered by the multiary λ-calculus, named λm.

In this talk we will give a brief overview of the multiary λ-calculus λm and of its subsystems, and present a mechanisation of these systems and their metatheory using the *Rocq Prover* and the *Autosubst* library. In particular, we will motivate our main mechanisation choices and highlight some differences when comparing to the original pen-and-paper definitions and proofs.

The logical essence of call-by-name CPS translations

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Abstract

The Curry-Howard isomorphism is a connection between logic and computation. Among many manifestations, there is the link between double-negation (DN) translations and continuation-passing style (CPS) translations. DN translations are interpretations of classical logic into intuitionistic logic. CPS is a compilation technique for high-level programming languages. We show that the surprising link can be refined. The "logical essence" of the CPS translation is a conversion of the source program, from the natural deduction format to the sequent calculus format, hence a change of deductive system without change of logic. The DN translation is an optional encoding of the sequent calculus. Composing the format conversion with the optional encoding gives back the CPS translation. We will restrict to the case when the source program follows the "call-by-name" evaluation strategy.

Characterization of the Drazin Index for Double Star Digraph Matrices

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Abstract

A fundamental challenge in spectral graph theory is identifying when matrices associated with graphs are invertible and formulating explicit expressions for their inverses. This challenge extends naturally to the study of pseudo-invertibility. In this work, we utilize the concept of Drazin invertibility—characterized by a specific nonsingular matrix—to ascertain the index of matrices linked to particular graph structures, notably double star digraphs. Our approach generalizes previous findings, offering a broader understanding of the conditions under which such matrices are Drazin invertible.

References

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Circulant matrices, t-products and Drazin inverses

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Abstract

In this exploratory talk, I will address (again) tensors and their algebraic properties, namely the t-product and the connection to circulant matrices, along with generalized invertibility. Their relation to hypergraphs will be also taken into account.