Book of Abstracts

of the ALC Meeting 2024

CMAT, University of Minho, Portugal

Contents

About the ALC Meeting

The ALC Meeting is organised by the research group Algebra, Logic and Computation (ALC) of the Centre of Mathematics (CMAT) at the University of Minho.

The purpose of the meeting is to bring together all the members of the ALC group, as well as their master and doctoral students, providing an opportunity to present new results and to discuss the latest research and developments in the scientific areas of interest to the group. It is intended to strengthen collaborations and to establish new connections within the group.

The 2024 Edition of the ALC Meeting

The ALC meeting 2024 takes place Friday, 20 December 2024 at the laboratory Lab4, Department of Mathematics, Gualtar Campus, University of Minho. The meeting is planned as an on site event; the talks will be shared via Zoom with participants from outside of Braga.

Organising Committee

The organising committee of the 2024 edition of the ALC meeting consists of the ALC members Carla Ferreira, René Gazzari and Luís Pinto.

Funding

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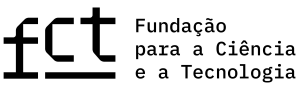

Schedule

09:15 - 09:20 *Welcome and Opening*

09:20 - 10:20 Invited Talk · Enide Andrade, *Graph Partition in ℓ_1-norm and Fiedler Theory*

10:20 - 10:40 Miguel Alves, *On the mechanisation of a fragment of the simply-typed lambda-calculus and its sequent calculus counterpart*

10:40 - 11:10 · *Coffee Break* ·

11:10 - 11:50 Rui Ralha, *Scratching the surface of ChatGPT*

11:50 - 12:10 Carlos Freitas, *Semigroups of transformations with an invariant set*

12:10 - 12:30 Ricardo Silva, *Non-Classicality of QRF Transformations in Spekkens' Stabilizer Toy Theory*

12:30 - 14:15 · *Lunch Break* ·

14:15 - 14:45 Eudes Costa, Paula Catarino and Elen Spreafico, *On the Incomplete Edouard and Incomplete Edouard-Lucas Numbers*

14:45 - 15:15 Angelos Bampounis, *Matchgate hierarchy: Deterministic gate teleportation in matchgate circuits*

15:15 - 15:45 Pedro Patrício, *EP products using Morita contexts*

15:45 - 16:20 · *Coffee Break* ·

16:20 - 16:50 René Gazzari, *Dealing with Fictional Numbers*

16:50 - 17:30 Robert E. Hartwig, *My mathematical life*

17:30 *Closing*

List of Abstracts

1 Enide Andrade. Graph Partition in ℓ_1-norm and Fiedler Theory

Partition problems in graphs are extremely important in applications, as shown in the Data Science and Machine Learning literature. One approach is spectral partitioning based on a Fiedler vector, i.e., an eigenvector corresponding to the second smallest eigenvalue $a(G)$ of the Laplacian matrix L_G of the graph G, [1]. This problem corresponds to the minimization of a quadratic form associated with L_G, under certain constraints involving the ℓ_2-norm. We present a similar problem, but using the ℓ_1-norm to measure distances. This leads to a new parameter $b(G)$ as the optimal value. We show that a well-known cut problem arises in this approach, namely the sparsest cut problem. We prove connectivity results and different bounds on this new parameter, related to Fiedler theory and show explicit expressions for $b(G)$ for trees, [2].

(Joint work with Geir Dahl, Department of Mathematics, University of Oslo.)

References

[1] M. Fiedler. A property of eigenvectors of nonnegative symmetric matrices and its application to graph theory, Czechoslov. Math. J., 25 (1975), 619-633.

[2] E. Andrade, G. Dahl. Combinatorial Fiedler Theory and Graph Partition, Linear Algebra Appl. 687, (2024), 229-251.

2 Miguel Alves. On the mechanisation of a fragment of the simply-typed lambda-calculus and its sequent calculus counterpart

It is a well-known result that the implicational fragment of natural deduction is isomorphic to the simply-typed lambda-calculus.

Gentzen's sequent calculus offers a richer space of proofs as compared to natural deduction. The question of finding fragments of sequent calculus and meaningful extensions of simply-typed lambda-calculus which still exhibit an isomorphic correspondence is a challenging question, which has been pursued in CMAT for the past 20 years.

The mechanisation of metatheory of such systems can be a good path to learn and clarify hidden details in "pen and paper" developments. In this talk we will illustrate some of the bureaucracy involved in such mechanisation. This will be exemplified with a well-behaved fragment of the simply-typed lambda-calculus (called the normal fragment) and its isomorphic sequent calculus counterpart.

3 Rui Ralha. Scratching the surface of ChatGPT

Artificial neural networks (ANN) are not new, they have been interesting scientists for several decades but only in recent years they have produced surprising results. This is because of the huge volume of data that is now available to train those networks and also due to the computing power of the parallel machines. The first release of ChatGPT happened two years ago and has had a significant impact in our lives. In this presentation we aim at explaining in simple terms the functioning of ANN behind ChatGPT.

4 Carlos Freitas. Semigroups of transformations with an invariant set

It is well-known that every semigroup can be embedded in a semigroup of transformations, and semigroups of transformations provide interesting examples of certain concepts in semigroup theory. Throughout this talk we will consider several semigroups of (linear) transformations that leave some set invariant and explore their structure.

5 Ricardo Silva. Non-Classicality of QRF Transformations in Spekkens' Stabilizer Toy Theory

Since classical mechanics, reference frame transformations have been essential tools for describing physical systems relative to other systems. Quantum reference frames extend this concept, exploring how such descriptions change when observed from the perspective of a quantum system. A key question arises: Are these transformations genuinely quantum, or can they be simulated within an epistemically restricted classical framework? In this presentation, we address this question by attempting to replicate these transformations within Spekkens' Stabilizer Toy Theory, examining their underlying motivation and implications.

6 Eudes Costa, Paula Catarino and Elen Spreafico. On the Incomplete Edouard and Incomplete Edouard-Lucas Numbers

We introduce two new sequences: the incomplete Edouard and the incomplete Edouard-Lucas numbers. In addition, we establish some properties, identities, and recurrence relations of these sequences. The relations of these new sequences with the, balancing, Lucas-balancing, incomplete balancing, and incomplete Lucas-balancing numbers are explored, and some new identities are provided. Moreover, the generating functions for the incomplete Edouard and incomplete Edouard-Lucas numbers are stated.

7 Angelos Bampounis. Matchgate hierarchy: Deterministic gate teleportation in matchgate circuits

A fruitful approach to understand the power of quantum computers is to explore restricted classes of computations which can be efficiently simulated by classical devices but regain full universal quantum computing power by the addition of extra resources. Two notable classes of computations, which are known not to be universal, are circuits built out of Clifford gates, and circuits built out of a special set of unitary two-qubit gates restricted to act on nearest-neighbour qubits, called matchgates. In order to achieve quantum universality, the gate sets of both models can be supplemented with special input states, known as 'magic' states. In the Clifford setting, non-stabilizer states suffice to promote Clifford gates to quantum universality. For matchgate circuits, it has been shown that any pure fermionic state that is not Gaussian can be used as a 'matchgate-magic' state.

The promotion of restricted class of circuits through quantum gate teleportation was put forward by Shor, and in more generality, by Gottesman and Chuang. The latter, in the same work, also introduced the Clifford hierarchy: an increasing family of sets of unitary gates that can be implemented deterministically using gate teleportation on stabiliser circuits. In this talk, I will present an analogous gate teleportation scheme for matchgate circuit computation, and a corresponding gate hierarchy, which we dub the matchgate hierarchy. I will briefly explain how the protocol deterministically implements any n-qubit gate in the hierarchy using adaptive matchgate circuits with magic states. In addition, I will present a neat characterization of gates in the matchgate hierarchy for two qubits, and for an arbitrary number of qubits I will propose a characterisation of the matchgate hierarchy by leveraging the fermionic Stone–von Neumann theorem.

Based on joint work with Rui Soares Barbosa and Nadish de Silva, available as https://arxiv.org/abs/2410.01887.

8 Pedro Patrício. EP products using Morita contexts

Range hermitian or EP matrices can be characterized in terms of generalized invertibility, and the problem of the product of complex EP matrices has been long studied. In this talk I will address the general case using a Morita context.

This is a joint work with Xavier Mary, Université Paris X, and is under development.

9 René Gazzari. Dealing with Fictional Numbers

Occasionally, mathematicians treat some of their numbers (or other kind of objects) as purely formal entities without (real) existence. This is the case in our paradigmatic example of the Gauss sum:

$$\sum_{k=0}^{n} k \;=\; \frac{1}{2} \cdot n \cdot (n+1)$$

The equation asserts something about natural numbers, but with recourse to the rational number $\frac{1}{2}$, which is not present (not existing) in the domain of discourse. Other examples, where mathematicians consider some numbers (more or less voluntarily) only as fictional, are briefly considered.

We address in our talk the problem of representing this phenomenon adequately. For this purpose, we developed the *logic of virtual extension*, a free logic; grammar, calculus and semantics of this logic are introduced.

10 Robert E. Hartwig. My mathematical life

We follow the odyssey through the "mountains of mathematics", that were met on my travels as a flowering mathematician from Adelaide. From phase-transitions to matrix theory, from g-inverses to partial orders and from algebra to ring theory. These were then complemented by cryptography, coding and intrusion detection.