Álgebra Linear C

folha viii — 2008/2009

1. Seja $T : V \to W$ uma transformação linear entre os espaços vectoriais V e W. Mostre que

   (a) $T(0) = 0$.

   (b) Para $v_1, ..., v_n \in V$, $\alpha_1, ..., \alpha_n \in \mathbb{K}$,
   $T(\alpha_1 v_1 + ... + \alpha_n v_n) = \alpha_1 T(v_1) + ... + \alpha_n (v_n)$.

2. Diga quais das aplicações seguintes, entre espaços vectoriais reais, são transformações lineares:

   (a) $F : \mathbb{R}^2 \to \mathbb{R}^3$ definida por $F(x, y) = (2x + y, x, y - x)$, $\forall (x, y) \in \mathbb{R}^2$.

   (b) $G : \mathbb{R}^3 \to \mathbb{R}^2$ definida por $G(x, y, z) = (y^2, y)$, $\forall (x, y, z) \in \mathbb{R}^3$.

   (c) $H : \mathbb{R}_2[x] \to \mathbb{R}^2$ definida por $H(ax^2 + bx + c) = (1, a + b)$, $\forall\ ax^2 + bx + c \in \mathbb{R}_2[x]$.

   (d) $T : \mathbb{R}^2 \to \mathbb{R}$ definida por $T(a, b) = 5a - 2b$, $\forall\ (a, b) \in \mathbb{R}^2$.

3. Diga, justificando, se existe

   (a) uma transformação linear $F : \mathbb{R}^3 \to \mathbb{R}^3$ tal que $F(1, 0, 0) = (0, 0, 1)$, $F(0, 0, 1) = (1, 0, 0)$, $F(7, 0, 14) = (0, 0, 7)$.

   (b) uma transformação linear $G : \mathbb{R}^2 \to \mathbb{R}^4$ tal que $G(-1, 2) = (0, 1, 2, 3)$, $G(2, -1) = (0, -1, -2, -3)$.

4. Considere as transformações lineares

   $F : \mathbb{R}^4 \to \mathbb{R}^3$ definida por $F(x, y, z, w) = (x - y, x + w, y + z)$, $\forall (x, y, z, w) \in \mathbb{R}^4$;

   $G : \mathbb{R}^4 \to \mathbb{R}^4$ definida por $G(x, y, z, w) = (x, x + z, -w, 2y + z)$, $\forall (x, y, z, w) \in \mathbb{R}^4$;

   $H : \mathbb{R}^3 \to \mathbb{R}^3$ definida por $H(1, 1, 1) = (2, 0, 1)$, $H(1, 1, 0) = (1, 0, -1)$, $H(1, 0, 0) = (0, 0, 2)$;

   $T : \mathbb{R}^3 \to \mathbb{R}^4$ definida por $T(x, y, z) = (x - y, 0, 0, x + y + z)$, $\forall (x, y, z) \in \mathbb{R}^3$.

   Determine, em relação às bases canónicas:
   i) $[F]$. ii) $[G]$. iii) $[H]$. iv) $[T]$. v) $[H \circ F]$.

5. Considere a transformação linear $F : \mathbb{R}^3 \to \mathbb{R}^3$ definida por

   $$F(x, y, z) = (x + y, 0, y - z),\ \forall (x, y, z) \in \mathbb{R}^3.$$

   Determine a matriz que representa F nas bases $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ e na canónica.

6. Seja $G : \mathbb{R}^3 \to \mathbb{R}^4$ a transformação linear definida por

$$G(1,0,0) = (1,0,1,0),\ G(0,1,0) = (0,1,-2,0),\ G(0,0,1) = (1,1,0,0).$$

   (a) Determine i) $G(2,3,1)$. ii) $G(-1,2,0)$.

   (b) Para as bases

   $$B_1 = \{(1,1,1),(0,1,1),(0,0,1)\} \text{ e } B_2 = \{(1,0,0,0),(1,1,0,0),(1,1,1,0),(1,1,1,1)\},$$

   determine a matriz que representa G nessas bases.

7. Considere, no espaço vectorial real $\mathbb{R}^3$, a base $\{(1,0,1),(1,1,0),(0,1,1)\}$ e o endomorfismo F definido por $F(1,0,1) = (1,-1,1)$, $F(1,1,0) = (2,1,1)$, $F(0,1,1) = (1,0,0)$.

   (a) Determine $[F]$, em relação à base canónica.

   (b) Determine $F(a,b,c)$, $\forall (a,b,c) \in \mathbb{R}^3$.

8. Considere as transformações lineares

   $F : \mathbb{R}^4 \to \mathbb{R}^3$ definida por $F(x,y,z,w) = (x-y, x+w, y+z)$, $\forall (x,y,z,w) \in \mathbb{R}^4$;

   $G : \mathbb{R}^4 \to \mathbb{R}^4$ definida por $G(x,y,z,w) = (x, x+z, -w, 2y+z)$, $\forall (x,y,z,w) \in \mathbb{R}^4$;

   $H : \mathbb{R}^3 \to \mathbb{R}^3$ definida por $H(1,1,1) = (2,0,1)$, $H(1,1,0) = (1,0,-1)$, $H(1,0,0) = (0,0,2)$;

   $T : \mathbb{R}^3 \to \mathbb{R}^4$ definida por $T(x,y,z) = (x-y, 0, 0, x+y+z)$, $\forall (x,y,z) \in R^3$;

   e as bases $B_1 = \{(0,0,0,1),(0,0,1,1),(0,1,1,1),(1,1,1,1)\}$ e

   $$B_2 = \{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\} \text{ de } \mathbb{R}^4;$$

   $B_1' = \{(1,1,1),(1,1,0),(1,0,0)\}$ e $B_2' = \{(1,0,0),(0,1,0),(0,0,1)\}$ de $\mathbb{R}^3$.

   Determine

   a) $[F]_{B_1,B_1'}$. b) $[F]_{B_1,B_2'}$. c) $[G]_{B_2,B_1}$. d) $[G]_{B_1,B_2}$

   e) $[H]_{B_1',B_2'}$. f) $[H]_{B_1',B_1'}$ g) $[T]_{B_2',B_2}$. h) $[T]_{B_1',B_1}$.