Chaotic lensing around boson stars and Kerr black holes with scalar hair

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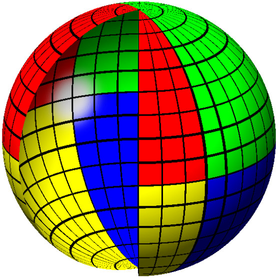
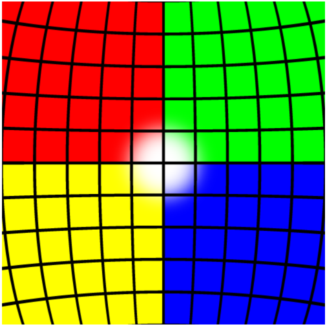
PRL **115**, 211102 (2015): C. Herdeiro, E. Radu, H. Rúnarsson

PRD **94**, 104023 (2016): *Ibid.* + J. Grover, A. Wittig

Observer setup

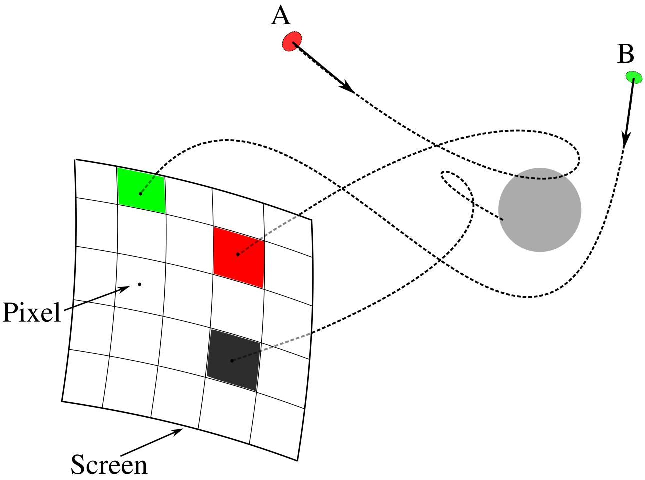




CQG 065002, Bohn+

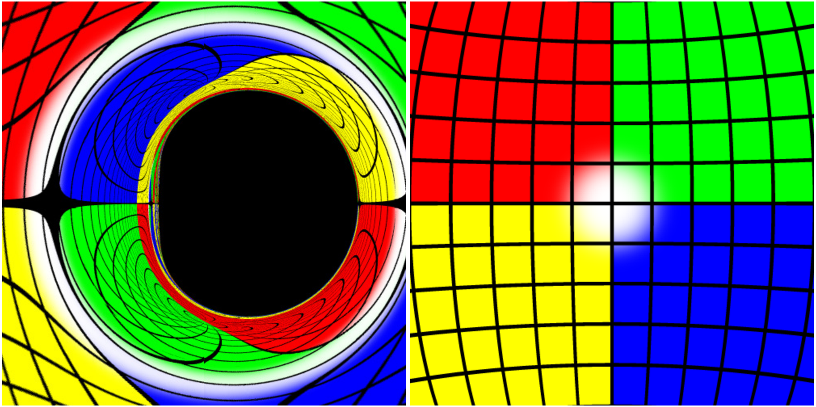
- A far-away colored sphere provides contrast.
- Compact objects will be placed in the center of the sphere.
- The observer is in the equatorial plane ($\theta = \pi/2$).

Backwards ray-tracing



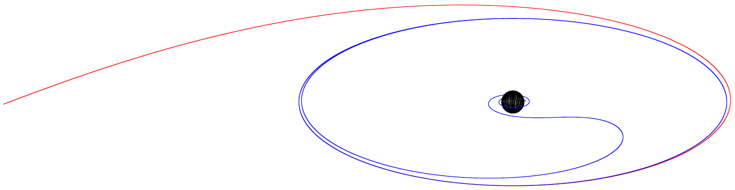
- Each image pixel sets an initial condition for a null geodesic.
- Shadow: set of conditions leading to geodesic infall into a BH.

Shadows of Kerr BHs



- The Kerr shadow is not symmetric due to rotation.
- Inside the Einstein ring $\rightarrow$ inverted copy of the celestial sphere.

Light rings (Kerr)

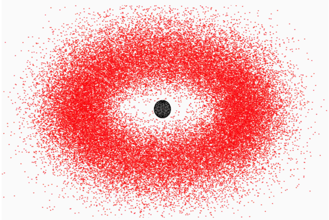


- *Light ring* $\rightarrow$ circular photon orbit in the equatorial plane.
- Kerr has *two* unstable light rings with opposite rotation.

Kerr BHs with scalar hair

Einstein-Klein-Gordon system

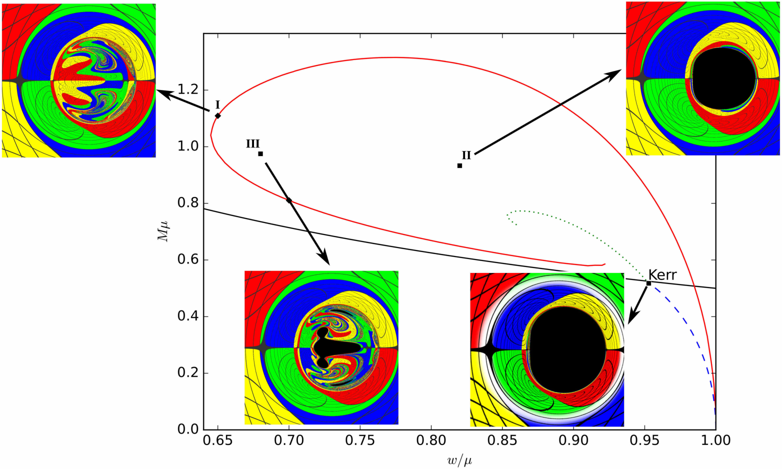
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \nabla_\nu \phi \nabla^\nu \phi^* - \mu^2 \phi^* \phi \right].$$



- Einstein's gravity minimally coupled to a complex scalar field ϕ.
- Stationary BH solutions exist, in equilibrium with ϕ.

Herdeiro & Radu: PRL 112.221101

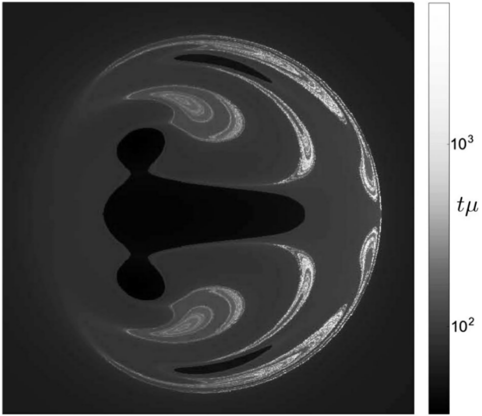
Solution space of Kerr BHs with scalar hair



- These hairy BHs are continuously connected to Boson Stars.

Cunha+ (PRL 115.211102)

Time delay function



- *Time delay function* defined as variation of coordinate t.
- Some geodesics require $100\times$ more integration time! Why?

Geodesic motion

- Null geodesics are described by the Hamiltonian $\mathcal{H}$:

$$\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0$$

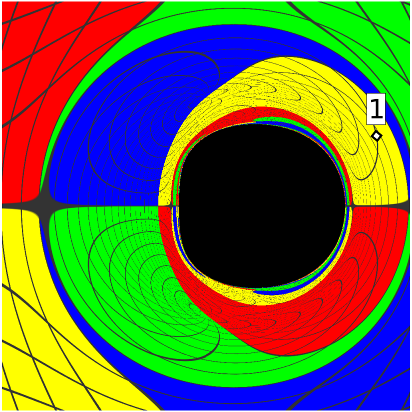
- In quasi-isotropic coordinates (t, r, θ, φ):

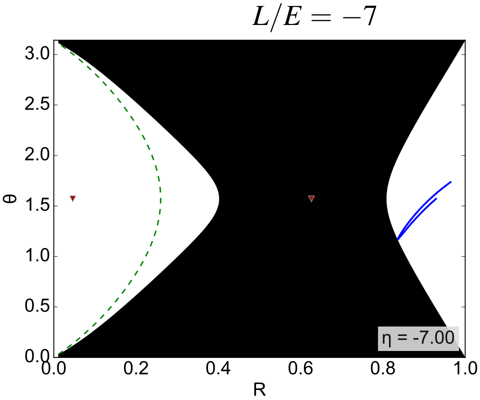
$$2\mathcal{H} = \underbrace{\left(g^{rr} {p_r}^2 + g^{\theta\theta} {p_\theta}^2\right)}_{T \geqslant 0} + \underbrace{\left(g^{tt} E^2 - 2g^{t\varphi} E\, L + g^{\varphi\varphi} L^2\right)}_{V \leqslant 0} = 0$$

- $2\mathcal{H}$ is a sum of a *kinetic* term T and a *potential* V.

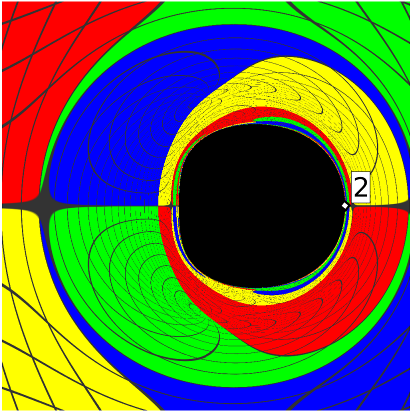
- E and L are the photon's energy and angular momentum.

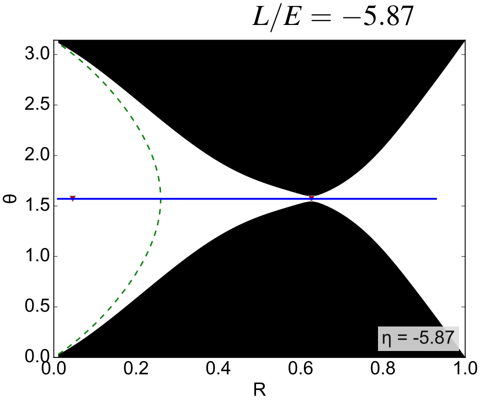
“Kerr-like” hairy BH





- $V > 0$ yields a *forbidden region* (black) in phase space (r, θ).





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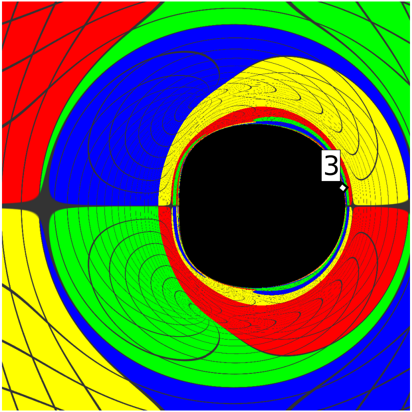

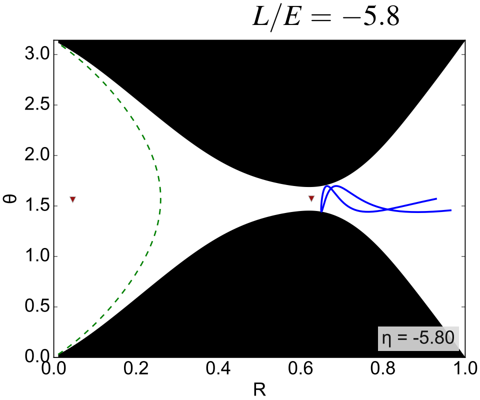

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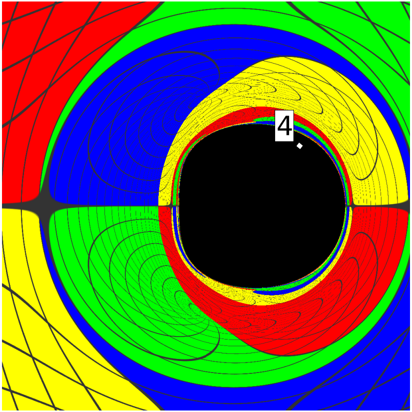

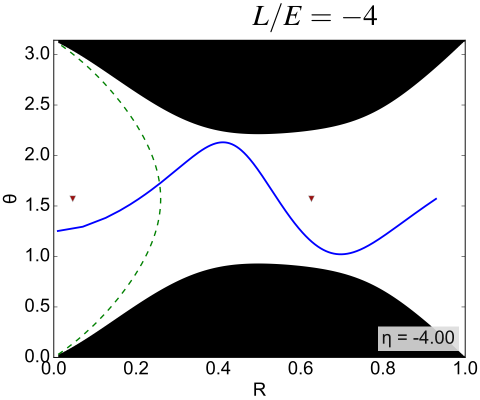

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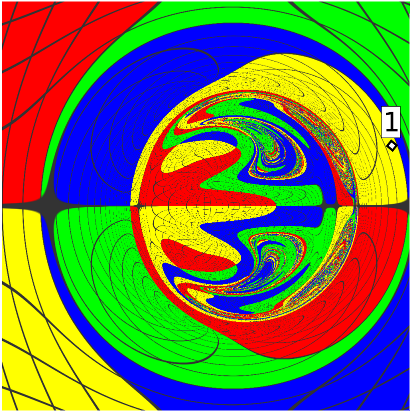
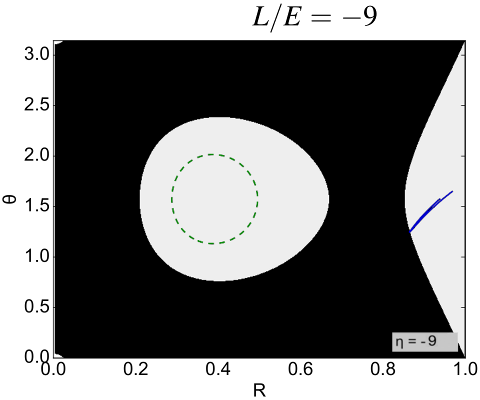
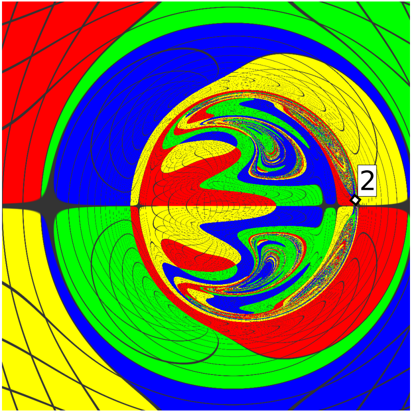
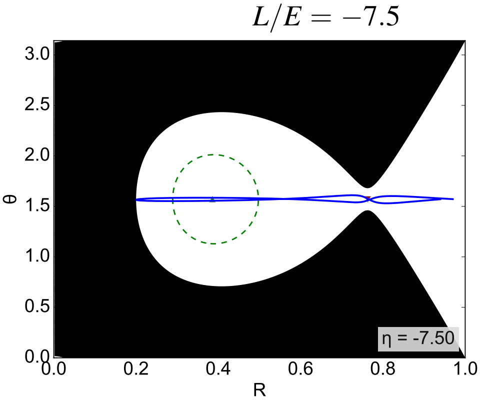
Boson Star



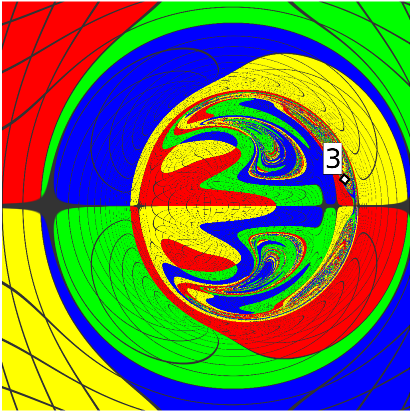


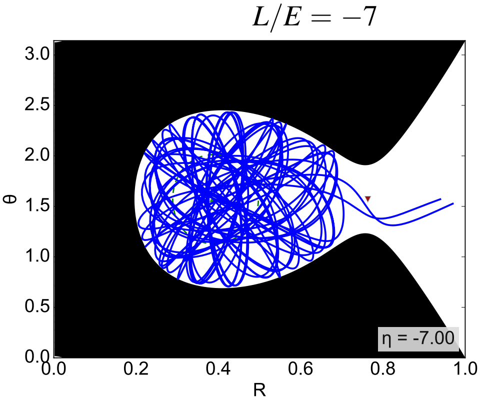
- We can have a disconnected (allowed) region $\rightarrow$ bounded orbits!





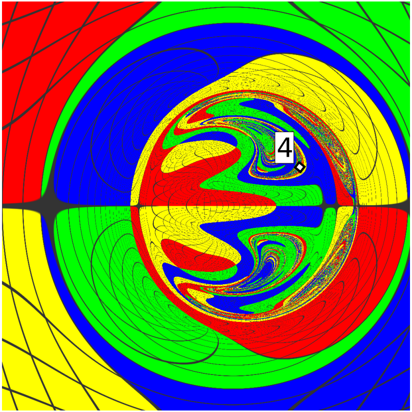
- By changing the impact parameter $\rightarrow$ small opening to a *pocket*.

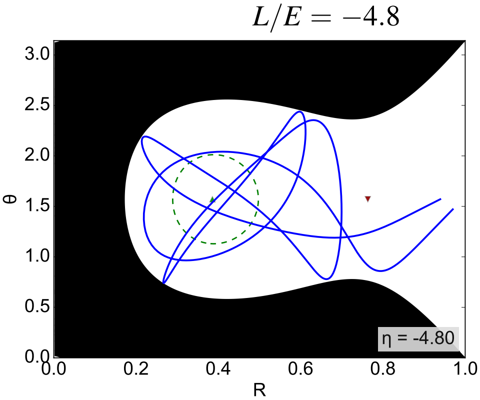




- This pocket can work as a *trapping region* for photon trajectories.

Boson Star





- This pocket can work as a *trapping region* for photon trajectories.

Geodesic motion

- V can be cast in the form:

$$g_{\varphi\varphi} + 2\eta\, g_{t\varphi} + \eta^2\, g_{tt} \geqslant 0$$

- The *impact parameter* $\eta = L/E$ is a constant of motion!

- Factorization leads to two 2D *effective potentials* $h_{\pm}$:

$$g_{tt}\,(\eta - h_+)(\eta - h_-) \geqslant 0$$

See also: CQG 175001, Dolan & Shipley

Why are $h_\pm$ useful?

Notice:

$$h_\pm = \eta \quad \Longrightarrow \quad p_r = p_\theta = 0$$

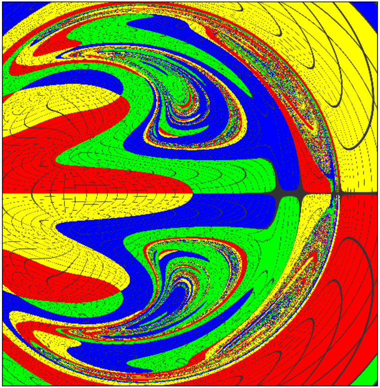
- The *contour lines* of $h_\pm$ give the forbidden region boundary!

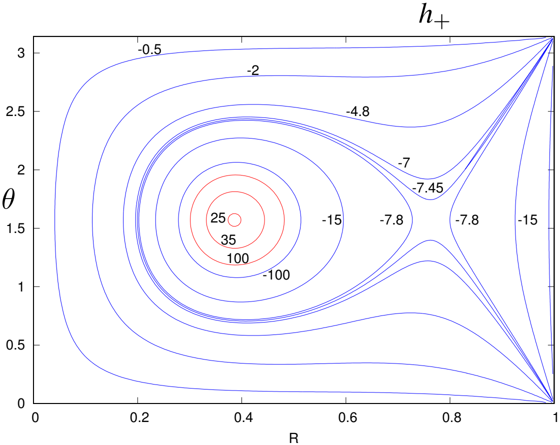
$$\boxed{h_\pm = \frac{-g_{t\varphi} \pm \sqrt{D}}{g_{tt}}} \quad \text{with} \quad D = g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}$$

- Light Rings (LRs) in the equatorial plane are *extrema* of $h_\pm$:

saddle point $\Longrightarrow$ *unstable* LR.
maximum $\Longrightarrow$ *stable* LR.

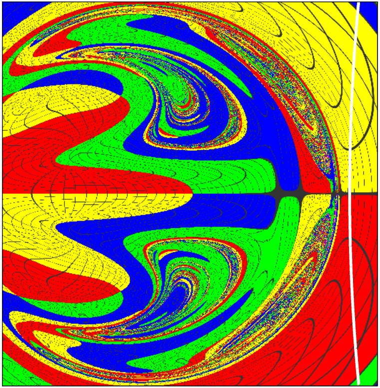
Example: Boson Star



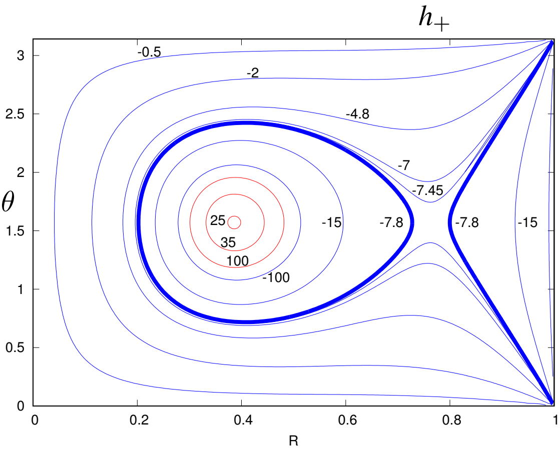


- The saddle point is an unstable Light Ring.

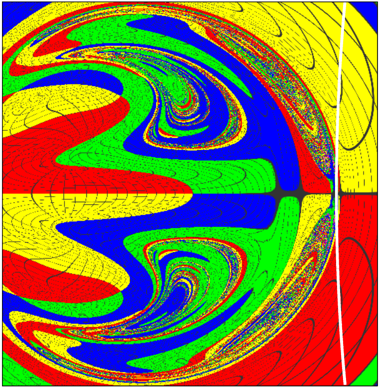
Example: Boson Star



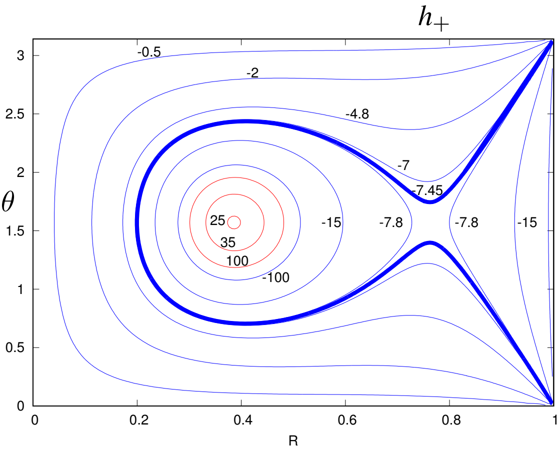
$\eta = -7.8$



- For some values of η we have an opening to a pocket.

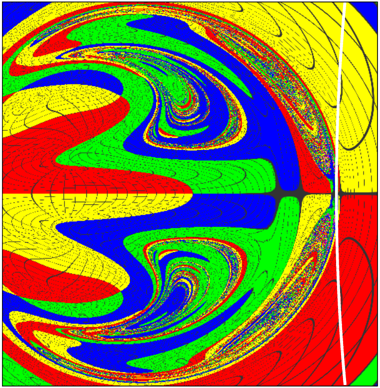


$\eta = -7.45$



- For some values of η we have an opening to a pocket.

Example: Boson Star



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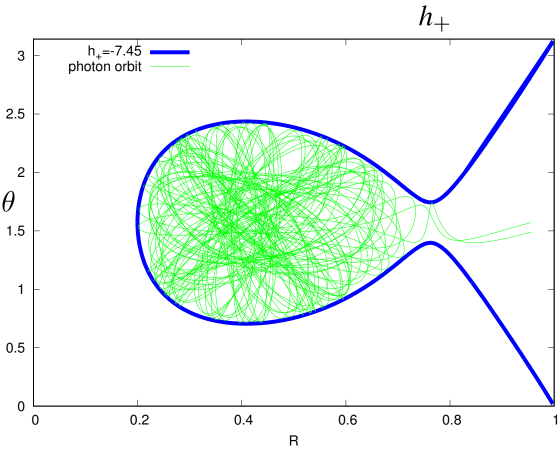
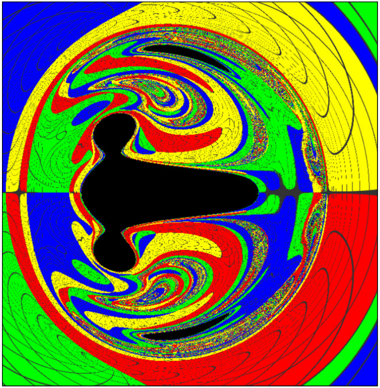


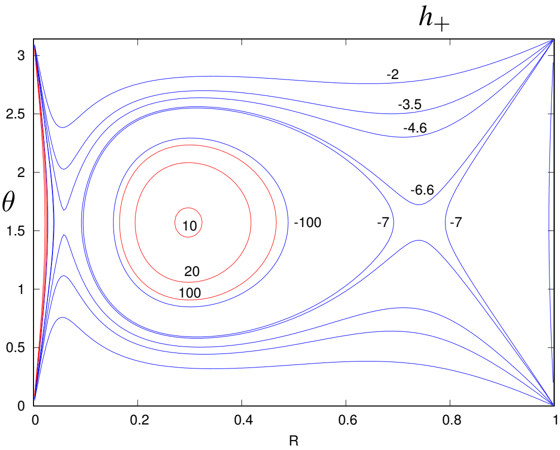
- The existence of a *quasi-bounded orbits* can lead to chaos!

Chaotic orbit (Boson Star)

- Chaotic orbit represented as if (r, θ, φ) were spherical coord.
- Most trajectory points exist inside a torus $S^1 \times S^1$.

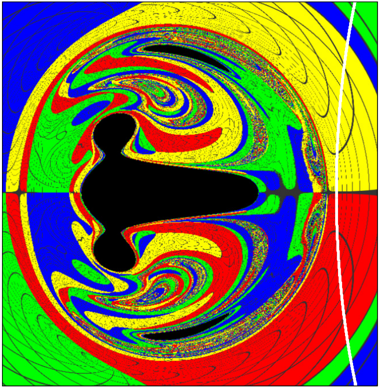
Example: Hairy Black Hole



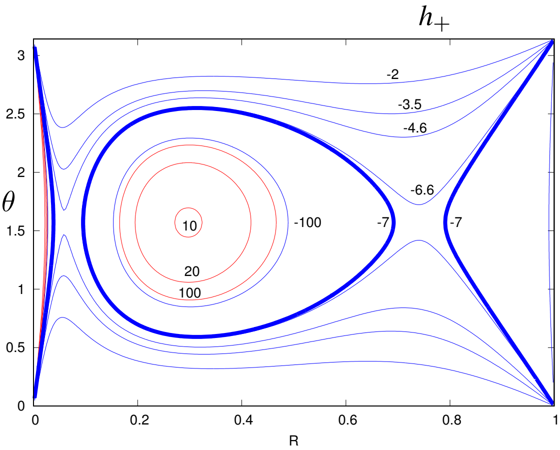


- There is an additional Light Ring close to the horizon.

Example: Hairy Black Hole

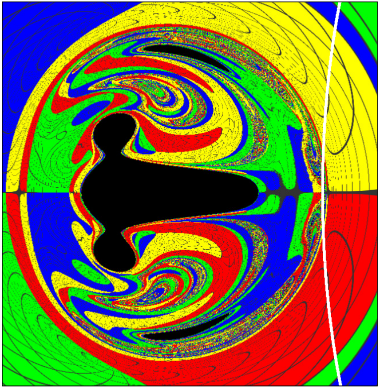


$\eta = -7$

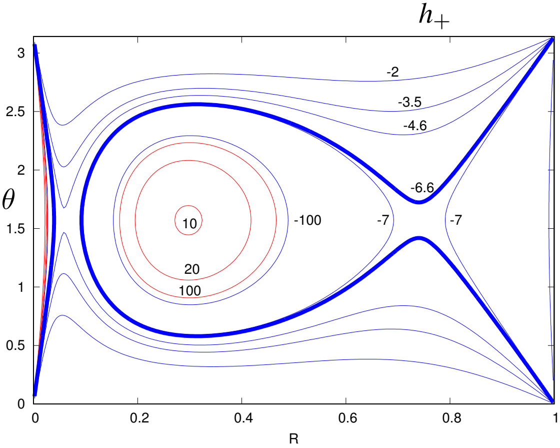


- For some values of η we have (again) an opening to a pocket.

Example: Hairy Black Hole



$\eta = -6.6$



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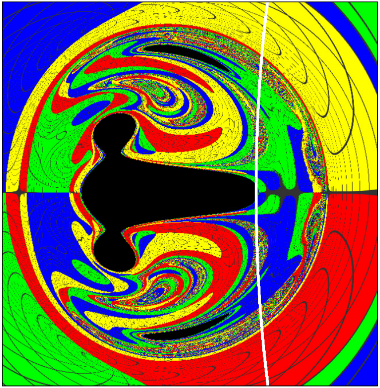
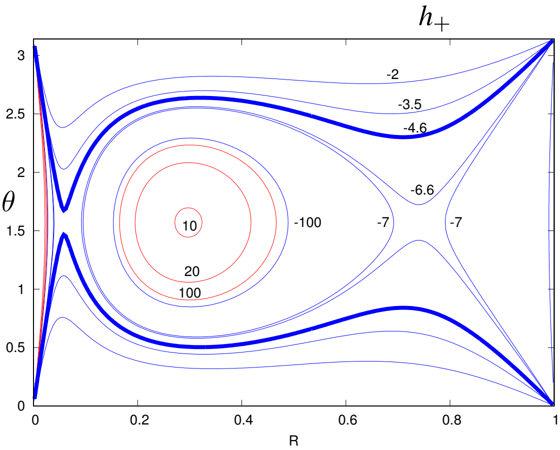
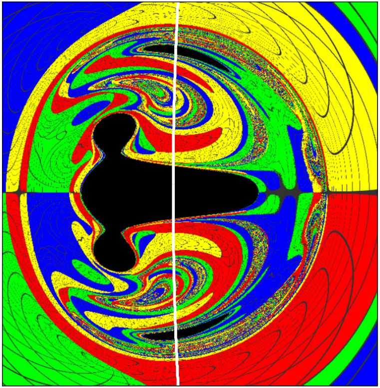


$\eta = -4.6$

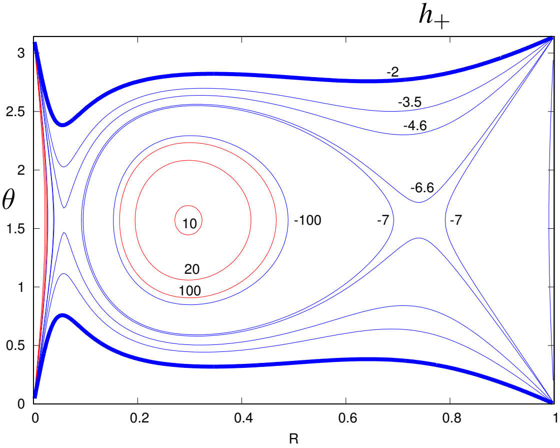


- An opening to the horizon forms a shadow.

Example: Hairy Black Hole



$\eta = -2$



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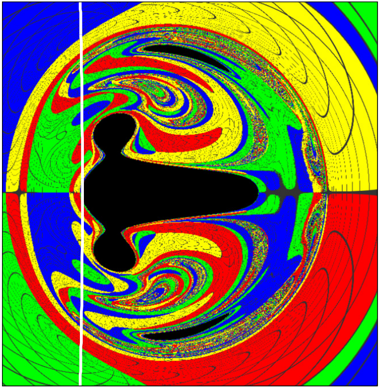
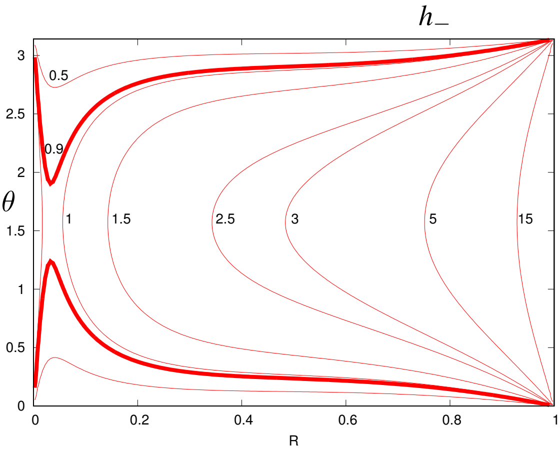
Example: Hairy Black Hole

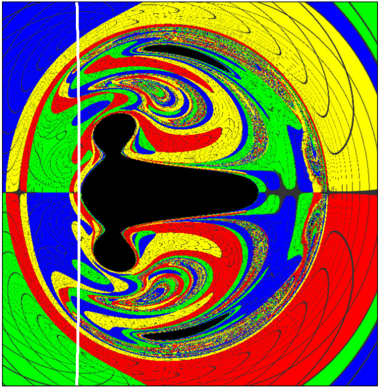


$\eta = 0.9$

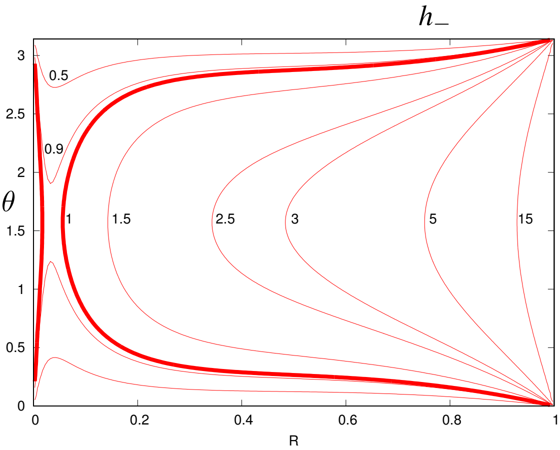


- By closing the opening to the horizon the shadow disappears.

Example: Hairy Black Hole



$\eta = 1$



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Turning points and Chaos

- The number of radial turning points n is correlated to chaos.
- For Kerr $n \leqslant 1$. A value $n > 1$ is a deviation from Kerr!

Turning points and Chaos

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Conclusions

- These hairy BH shadows are distinct from Kerr's.
- The existence of *quasi-bounded orbits* can lead to chaos !
- Pockets are connected to the existence of a *stable* light ring.
- Some features can be understood via the $h_{\pm}$ potentials.
- Method generic to stationary and axially symmetric spacetimes.

Acknowledgements

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