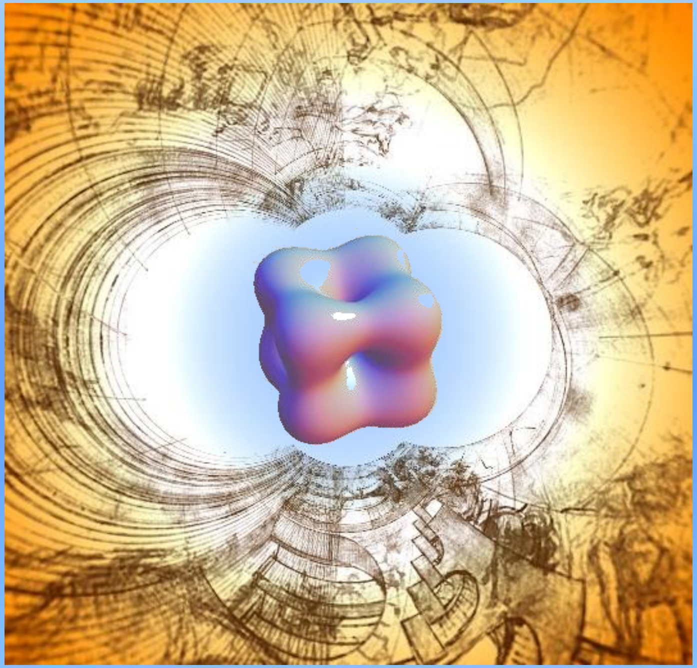
IX Black Holes Workshop , Guimarães, 19.12.2016

Einstein–Maxwell–Anti-de-Sitter solitons and black holes



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based mainly on

Physical Review Letters 117 (2014) 221102

Phys. Lett. B749 (2015) 393

with ***C. Herdeiro*** (Aveiro University)

Introduction

- ***Einstein-Maxwell system***: classic subject in General Relativity
  (here, four dimensions only)

•**Asymptotically flat spacetime:** ***remarkable simple picture***

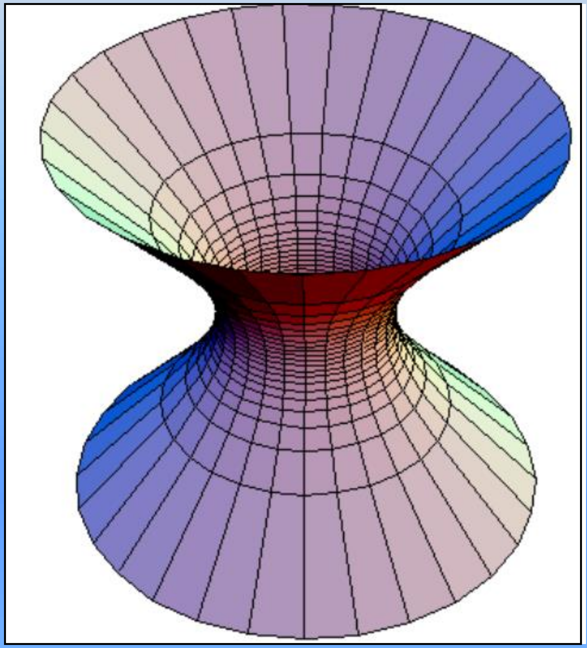
Israel's theorem (1968):
"In electrovacuum, a single, static, Black Hole is spherically symmetric and described by only two parameters (M,Q)"

⇕

Reissner-Nordstrom Black Hole

- **no (Einstein)-Maxwell solitons**
  (Lichnerowicz-type theorems)

what about (globally) AdS_4 spacetime?

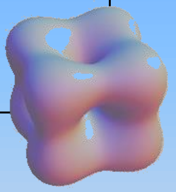


- RN ⟶ RNAdS Black Hole *(only solution known so far)*
- however, no uniqueness results *(no Israel-type theorem)*
- also, no solution generation techniques

the message here:

known EM Black Holes

new Black Holes
+
solitons



AdS: ***non-Reissner-Nordstom configurations without asymptotically flat counterparts***

Electrostatics in (globally) AdS spacetime

fixed globally AdS background:

$$\Lambda \equiv -3/L^2 < 0$$

$$ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\ , \quad \text{where} \quad N(r) \equiv 1 + \frac{r^2}{L^2}\ .$$

Maxwell equations

$$\nabla_\mu F^{\mu\nu} = 0$$

U(1) potential

$$A = V(r,\theta,\varphi)dt$$

$$V(r,\theta,\phi) = \sum_{\ell\geq 1}\sum_{m=-\ell}^{m=\ell} c_{\ell m} R_\ell(r) Y_{\ell m}(\theta,\varphi)$$

$$\frac{d}{dr}\left(r^2\frac{dR_\ell(r)}{dr}\right) = \frac{\ell(\ell+1)}{N(r)}R_\ell$$

$$\frac{d}{dr}\left(r^2\frac{dR_\ell(r)}{dr}\right) = \frac{\ell(\ell+1)}{N(r)}R_\ell$$

Minkowski spacetime::

$$N(r) = 1$$

$$R_\ell(r) = c_1 r^\ell + \frac{c_2}{r^{\ell+1}}$$

$$\frac{d}{dr}\left(r^2\frac{dR_\ell(r)}{dr}\right)=\frac{\ell(\ell+1)}{N(r)}R_\ell$$

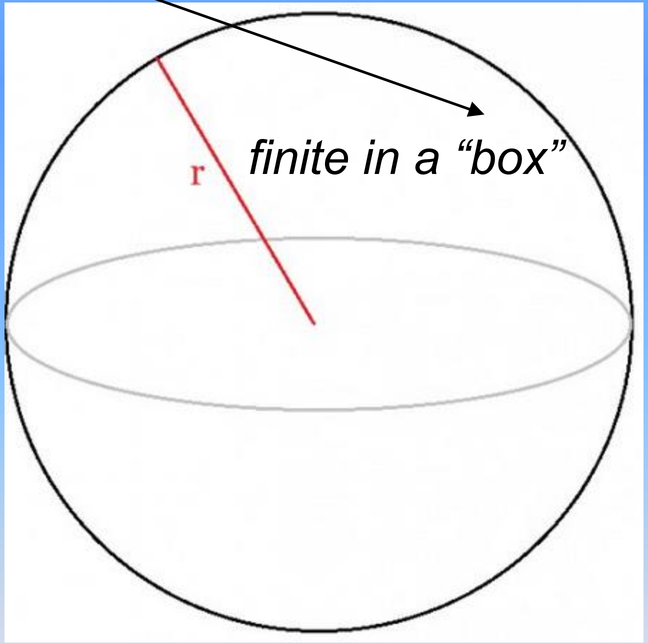
***Minkowski spacetime:*:** $N(r)=1$

$$R_\ell(r)=c_1 r^\ell+\frac{c_2}{r^{\ell+1}}$$

AdS spacetime: $N(r)=1+\frac{r^2}{L^2}$

$$R_\ell(r)=c_1 r^\ell+\frac{c_2}{r^{\ell+1}}$$

***is regularized*!**



flat ***AdS***

$$R_\ell(r) = r^\ell$$

$$R_\ell(r) = \frac{\Gamma(\frac{1+\ell}{2})\Gamma(\frac{3+\ell}{2})}{\sqrt{\pi}\Gamma(\frac{3}{2}+\ell)} \frac{r^\ell}{L^\ell} \, {}_2F_1\left(\frac{1+\ell}{2}, \frac{\ell}{2}; \frac{3}{2}+\ell; -\frac{r^2}{L^2}\right)$$

new solution!

$r \to 0$

$$R_\ell(r) = \frac{\Gamma\left(\frac{1+\ell}{2}\right)\Gamma\left(\frac{3+\ell}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{3}{2}+\ell\right)} \left(\frac{r}{L}\right)^\ell + \dots .$$

$r \to \infty$

$$R_\ell(r) = 1 - \frac{2\Gamma\left(\frac{1+\ell}{2}\right)\Gamma\left(\frac{3+\ell}{2}\right)}{\Gamma\left(1+\frac{\ell}{2}\right)\Gamma\left(\frac{\ell}{2}\right)} \frac{L}{r} + \dots .$$

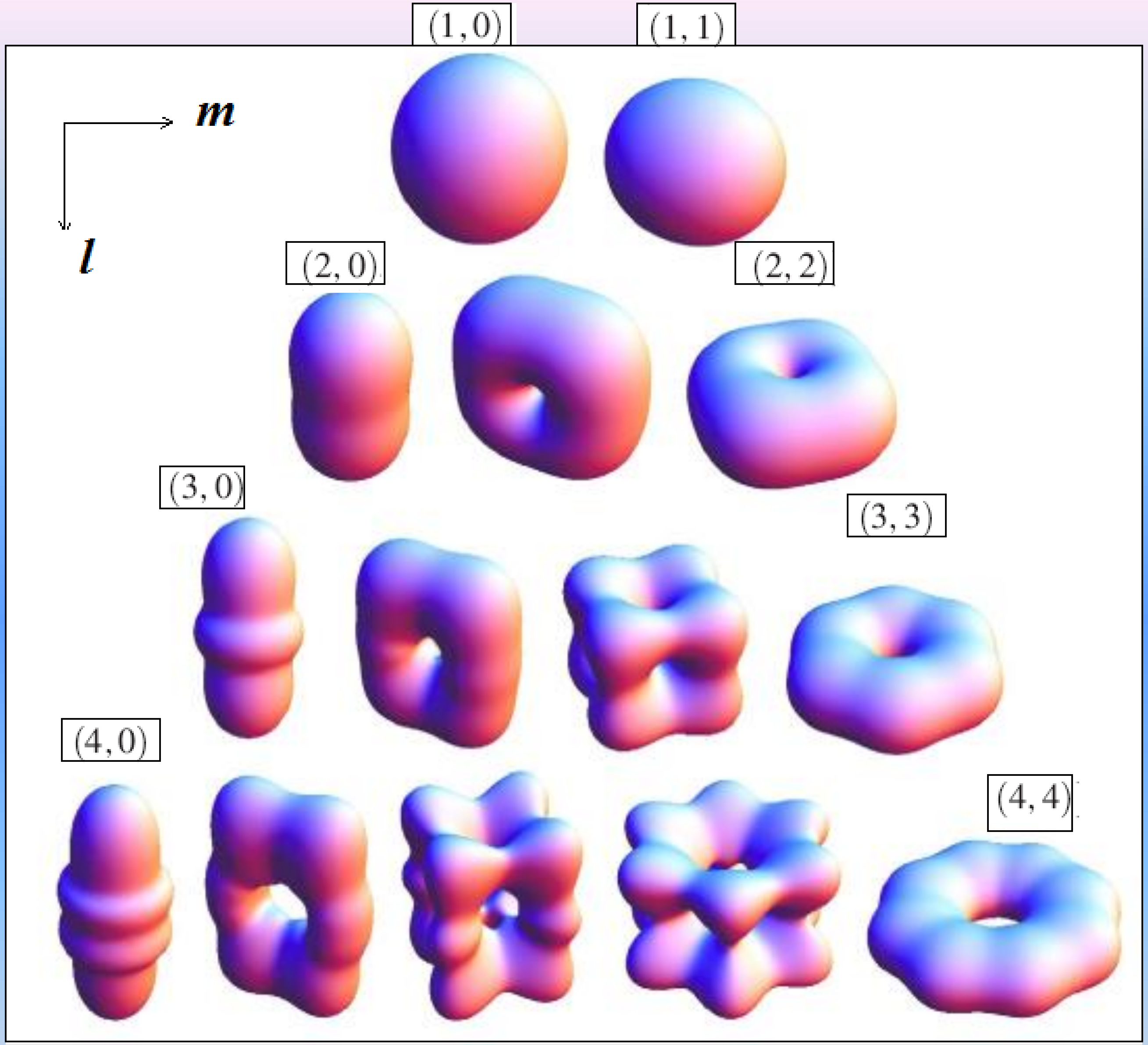
the *l=0* mode is missing! → **no *net* electric charge**

(but *nonzero* electric charge density)

$\ell \geq 1$

smooth, arbitrary superposition of multipoles

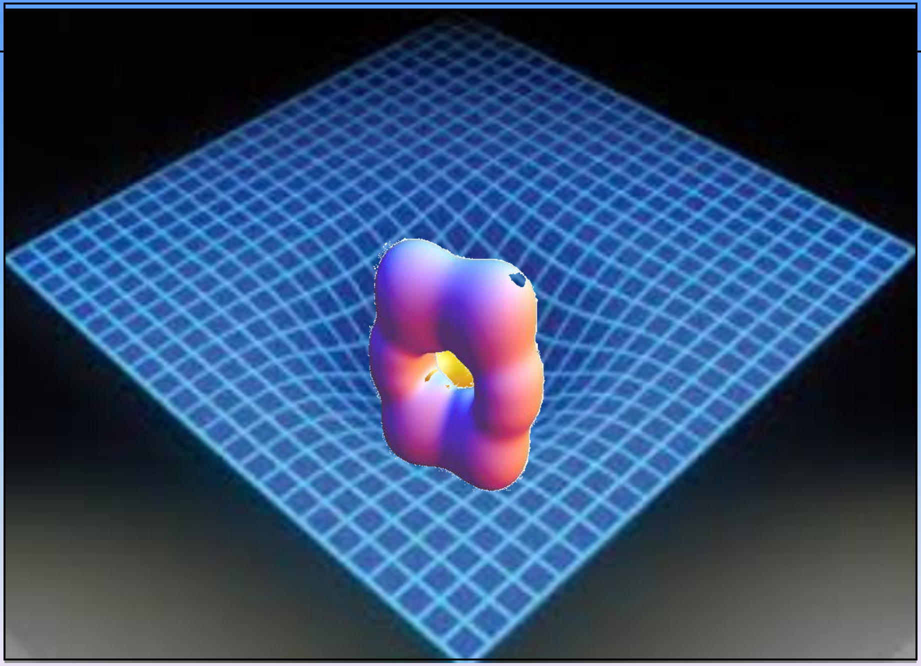
$$E_\ell = L \frac{\Gamma(\frac{1+\ell}{2})\Gamma(\frac{3+\ell}{2})}{\Gamma(1+\frac{\ell}{2})\Gamma(\frac{\ell}{2})}$$



Examples of surfaces of constant energy density for the Maxwell–AdS regular electric multipoles global AdS, unlike global Minkowski, **admits everywhere regular, finite energy electric fields** for all multipoles *except* the monopole

now, the setup is clear:

i) the solutions should survive when including backreaction → Einstein-Maxwell-AdS solitons with discrete symmetries only

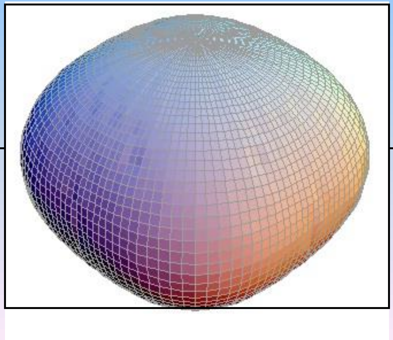


global AdS, unlike global Minkowski, **admits everywhere regular, finite energy electric fields** for all multipoles *except* the monopole

now, the setup is clear:

i) the solutions should survive when including backreaction → Einstein-Maxwell-AdS solitons with discrete symmetries only

ii) moreover, when gravitating solitons exist in a given model, bound states of such solitons with an event horizon can typically be constructed → Static Einstein-Maxwell-AdS black holes with discrete symmetries only



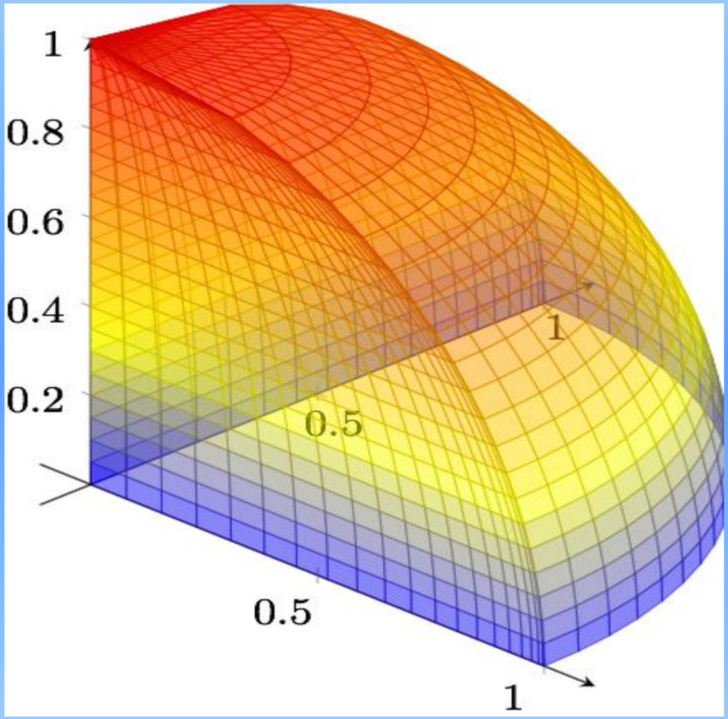
The generic solutions are static and possess discrete symmetries only

(r, θ, φ) grid

$N_r \times N_\theta \times N_\varphi$ points

single Killing vector $\partial/\partial t$

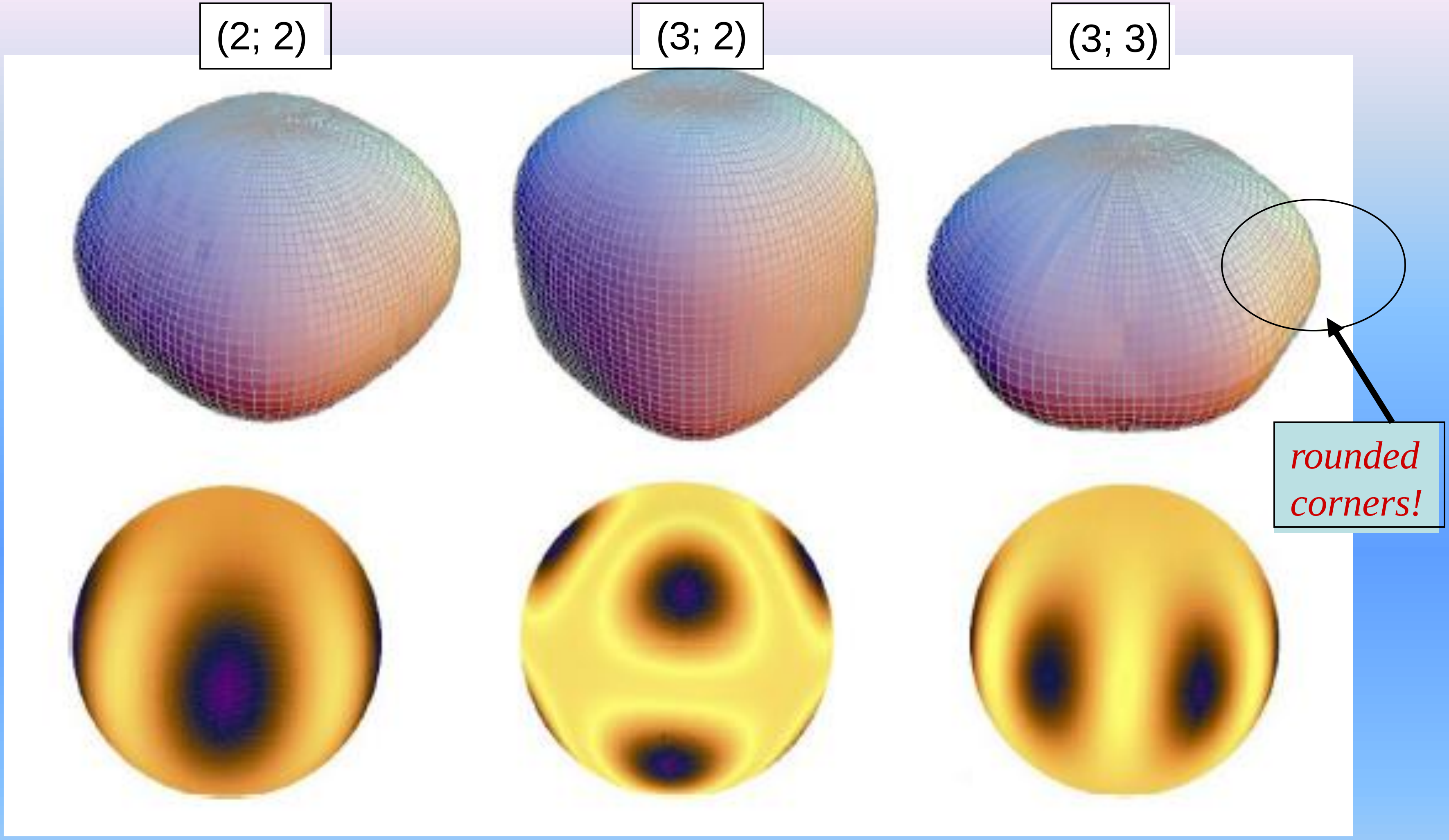
first, fully nonlinear, codimension-3 solutions in GR



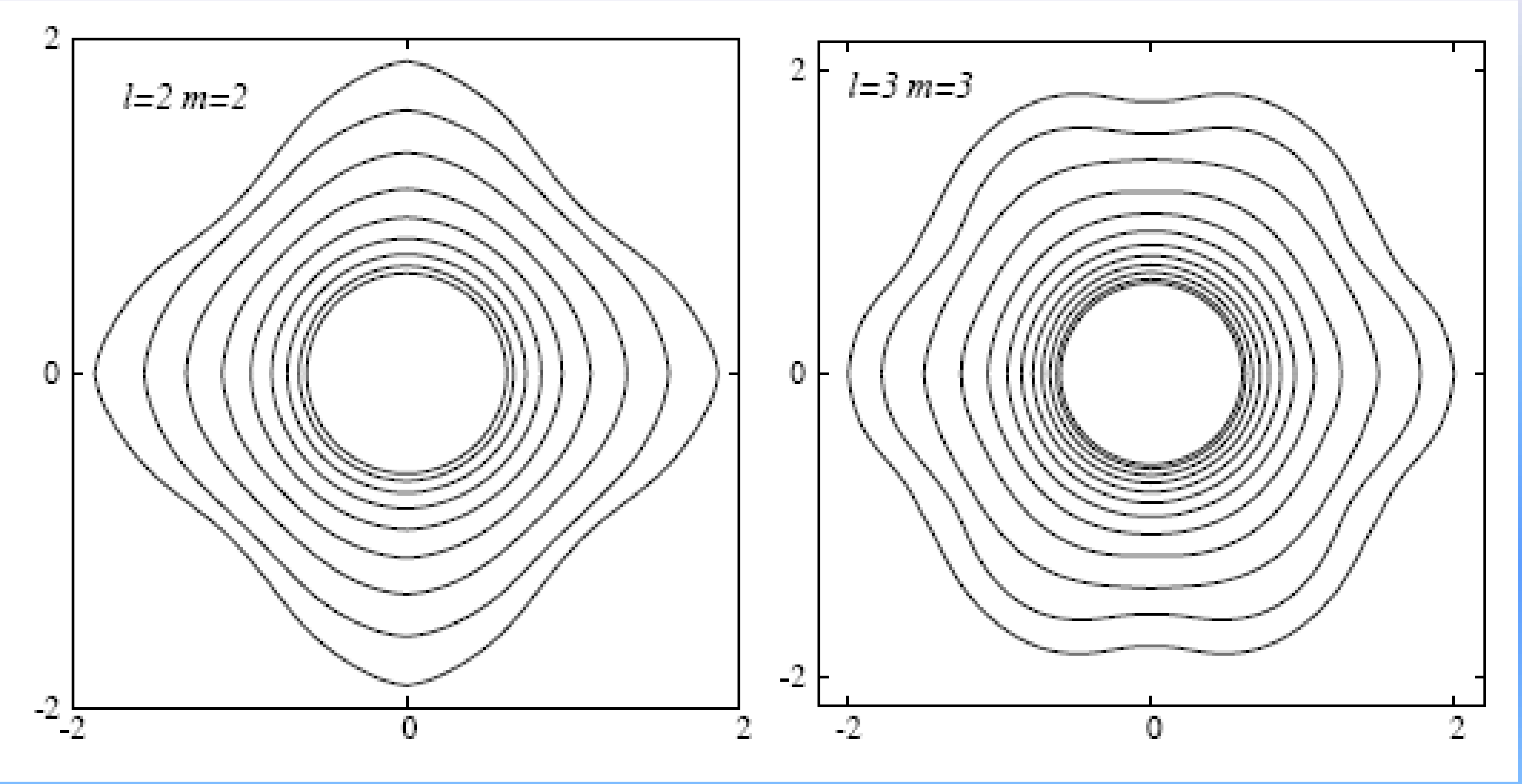
the solutions are regular on and outside the horizon

- ***no exact solutions:*** however, perturbative results
- ***existence proof:***

P. Chrusciel, E. Delay **arXiv:1612.00281** [math.DG]



examples of ***isometric embeddings*** for the horizon of AdS-electrovacuum BHs (top), together with their ***horizon Ricci scalar*** (bottom).

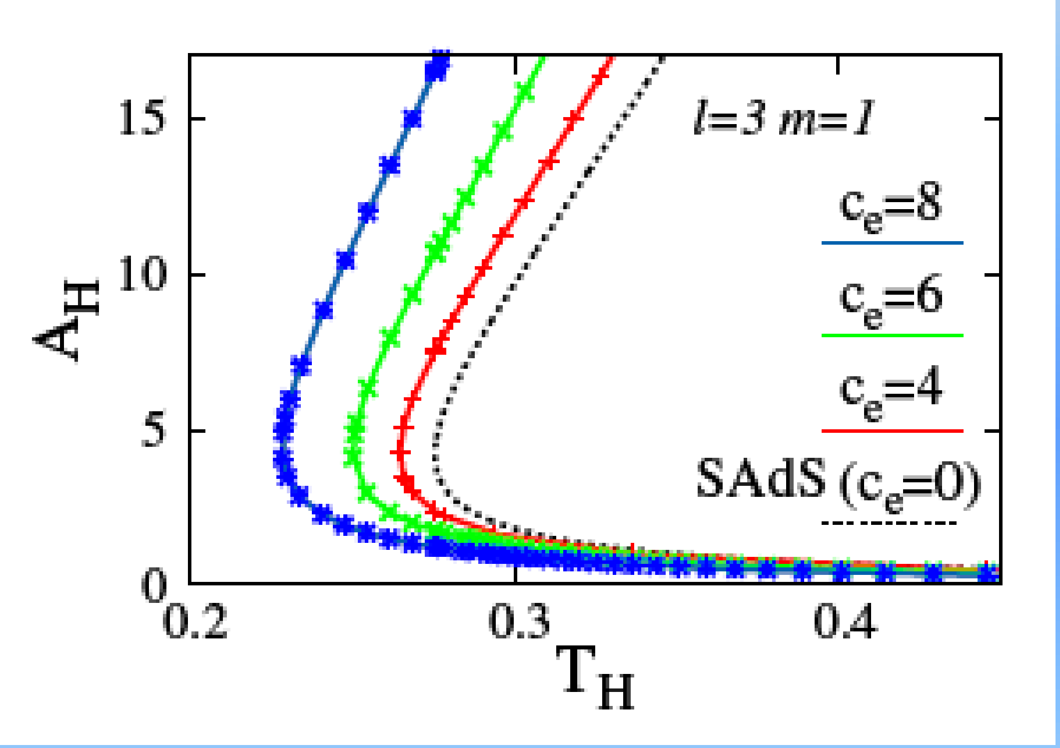


Equatorial slices for isometric embeddings of the horizons of AdS-electrovacuum BHs with different boundary data. The BHs have the same temperature and increasing values of the parameter c_e, starting with $c_e = 0$ (center).

$$V = c_e Y_{\ell m}(\theta, \varphi)$$

No electric flux at infinity ⇒ the SAdS pattern is recovered

no extremal Black Holes!



Horizon area vs: temperature for (*3; 1*) BHs

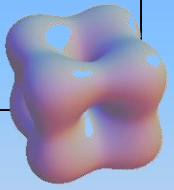
Possible interpretation: distorted SAdS Black Holes with electric multipoles

Outlook

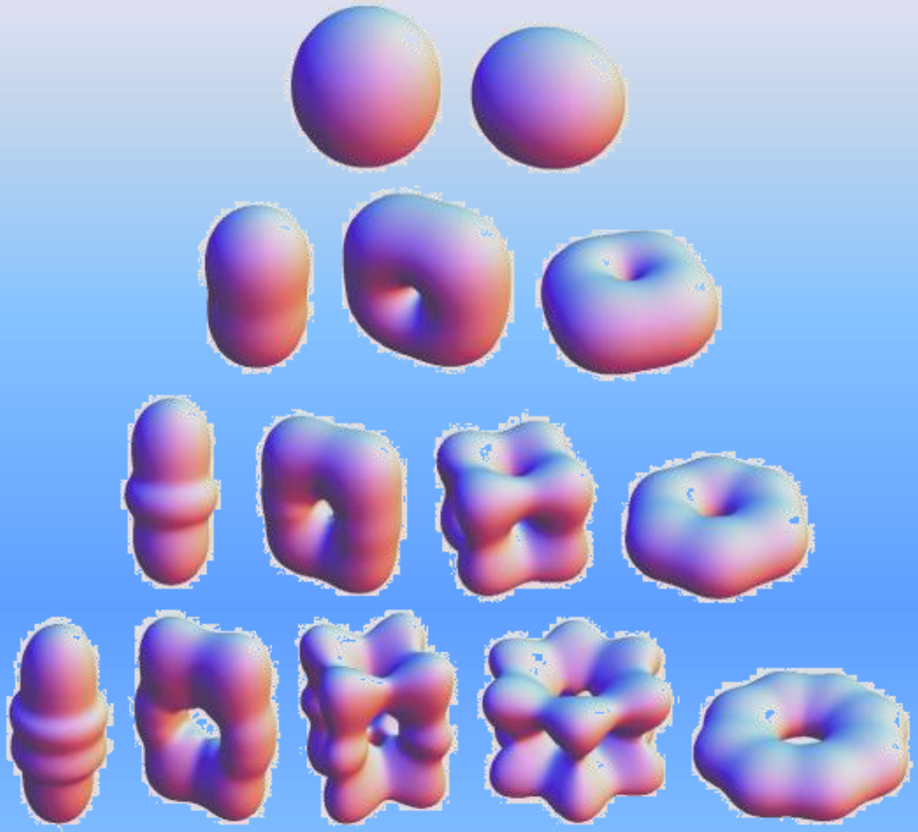
main message:

known EM Black Holes

new Black Holes
+
solitons



- static BHs in AdS electrovacuum can have an arbitrary electric multipole structure
- nontrivial solitonic limit



Muito obrigado pela vossa atenção!