Quantum *vs* classical:

non-locality, contextuality, and informatic advantage

Rui Soares Barbosa

DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF OXFORD

rui.soares.barbosa@cs.ox.ac.uk

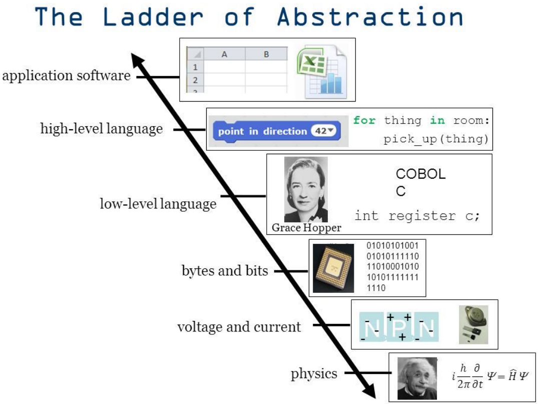
Q DAYS 2019
CMAT – Centro de Matemática, Universidade do Minho
12th April 2019

Motivation

- Computers are physical machines

Motivation

- Computers are physical machines
- But Computer Science tends to ignore this ...



Indeed, therein lies its great strength!

Indeed, therein lies its great strength!

Motivation

- use **quantum resources** for information-processing tasks
- delineate the scope of **quantum advantage**

Motivation

- use **quantum resources** for information-processing tasks
- delineate the scope of **quantum advantage**
- What non-classical features of quantum mechanics are responsible for quantum advantage?
  - identify the essential structure
  - theory-independent

Einstein–Podolsky–Rosen

- ‘Spooky’ action at a distance.

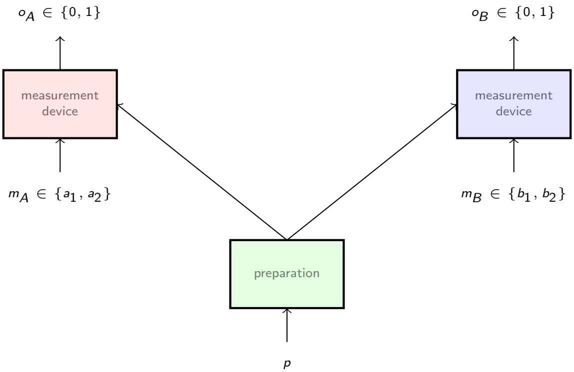
Einstein–Podolsky–Rosen

- ‘Spooky’ action at a distance.
- But is this so spooky?

Einstein–Podolsky–Rosen

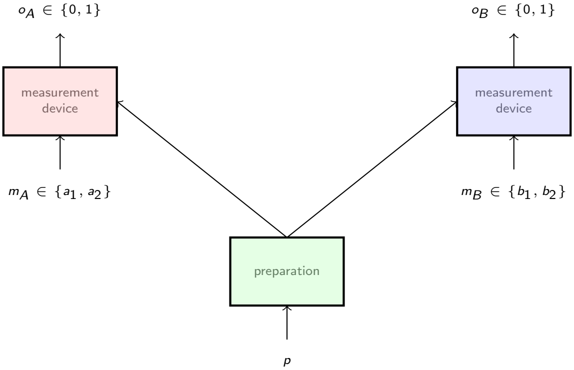
- 'Spooky' action at a distance.
- But is this so spooky?
- EPR conclusion: QM is incomplete

Empirical data



Empirical data

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_2	$1/8$	$3/8$	$3/8$	$1/8$



A simple observation

(Abramsky–Hardy)

- Propositional formulae $\phi_1, \ldots, \phi_N$

A simple observation

(Abramsky–Hardy)

- Propositional formulae $\phi_1, \ldots, \phi_N$
- $p_i := \mathrm{Prob}(\phi_i)$

A simple observation

(Abramsky–Hardy)

- Propositional formulae $\phi_1, \ldots, \phi_N$
- $p_i := \mathrm{Prob}(\phi_i)$
- Not simultaneously satisfiable, hence

$$\mathrm{Prob}(\bigwedge \phi_i) = 0$$

A simple observation

(Abramsky–Hardy)

- Propositional formulae $\phi_1, \ldots, \phi_N$
- $p_i := \mathrm{Prob}(\phi_i)$
- Not simultaneously satisfiable, hence

$$\mathrm{Prob}(\bigwedge \phi_i) = 0$$

- Using elementary logic and probability:

$$\begin{aligned} 1 = \mathrm{Prob}(\neg \bigwedge \phi_i) &= \mathrm{Prob}(\bigvee \neg\phi_i) \\ &\leq \sum_{i=1}^{N} \mathrm{Prob}(\neg\phi_i) = \sum_{i=1}^{N}(1 - p_i) = N - \sum_{i=1}^{N} p_i. \end{aligned}$$

A simple observation

(Abramsky–Hardy)

- Propositional formulae $\phi_1, \ldots, \phi_N$
- $p_i := \mathrm{Prob}(\phi_i)$
- Not simultaneously satisfiable, hence

$$\mathrm{Prob}(\bigwedge \phi_i) = 0$$

- Using elementary logic and probability:

$$\begin{aligned} 1 = \mathrm{Prob}(\neg \bigwedge \phi_i) &= \mathrm{Prob}(\bigvee \neg\phi_i) \\ &\leq \sum_{i=1}^{N} \mathrm{Prob}(\neg\phi_i) = \sum_{i=1}^{N}(1 - p_i) = N - \sum_{i=1}^{N} p_i . \end{aligned}$$

- Hence, $\sum_{i=1}^{N} p_i \leq N - 1$.

Analysis of the Bell table

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_2	$1/8$	$3/8$	$3/8$	$1/8$

Analysis of the Bell table

A	B	$(0, 0)$	$(0, 1)$	$(1, 0)$	$(1, 1)$
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_2	$1/8$	$3/8$	$3/8$	$1/8$

$$\phi_1 = a_1 \leftrightarrow b_1$$
$$\phi_2 = a_1 \leftrightarrow b_2$$
$$\phi_3 = a_2 \leftrightarrow b_1$$
$$\phi_4 = a_2 \oplus b_2$$

Analysis of the Bell table

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_2	$1/8$	$3/8$	$3/8$	$1/8$

$$\phi_1 = a_1 \leftrightarrow b_1$$
$$\phi_2 = a_1 \leftrightarrow b_2$$
$$\phi_3 = a_2 \leftrightarrow b_1$$
$$\phi_4 = a_2 \oplus b_2$$

These formulae are contradictory.

Analysis of the Bell table

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_2	$1/8$	$3/8$	$3/8$	$1/8$

$$\phi_1 = a_1 \leftrightarrow b_1$$
$$\phi_2 = a_1 \leftrightarrow b_2$$
$$\phi_3 = a_2 \leftrightarrow b_1$$
$$\phi_4 = a_2 \oplus b_2$$

These formulae are contradictory.
But

$$p_1 + p_2 + p_3 + p_4 = 3.25$$

Analysis of the Bell table

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_2	$1/8$	$3/8$	$3/8$	$1/8$

$$\phi_1 = a_1 \leftrightarrow b_1$$
$$\phi_2 = a_1 \leftrightarrow b_2$$
$$\phi_3 = a_2 \leftrightarrow b_1$$
$$\phi_4 = a_2 \oplus b_2$$

These formulae are contradictory.
But

$$p_1 + p_2 + p_3 + p_4 = 3.25$$

The inequality is violated by $1/4$.

Contextuality

- But the Bell table can be realised in the real world.

Contextuality

- But the Bell table can be realised in the real world.
- What was our unwarranted assumption?

Contextuality

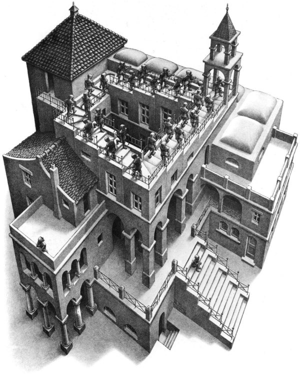
- But the Bell table can be realised in the real world.
- What was our unwarranted assumption?
- That all variables could *in principle* be observed simultaneously.

Snapshots

- Not all properties of a quantum system may be observed at once.
- Jointly measurable observables provide **partial, classical snapshots**.

Snapshots

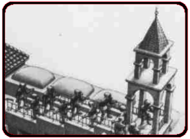
- Not all properties of a quantum system may be observed at once.
- Jointly measurable observables provide **partial, classical snapshots**.



M. C. Escher, *Ascending and Descending*

Snapshots

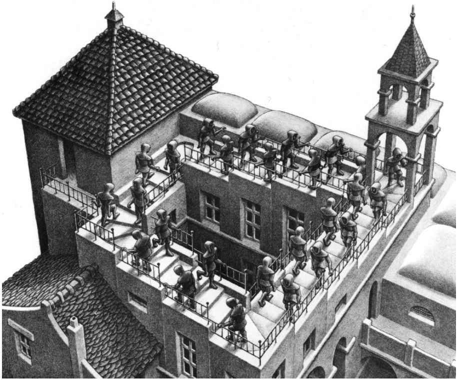
- Not all properties of a quantum system may be observed at once.
- Jointly measurable observables provide **partial, classical snapshots**.



Local consistency

Snapshots

- Not all properties of a quantum system may be observed at once.
- Jointly measurable observables provide **partial, classical snapshots**.



Local consistency *vs* **Global inconsistency**

Abramsky–Brandenburger framework

Measurement scenario $\langle X, \mathcal{M}, O \rangle$:

- X is a finite set of measurements or variables
- O is a finite set of outcomes or values
- $\mathcal{M}$ is a cover of X, indicating **joint measurability** (contexts)

Abramsky–Brandenburger framework

Measurement scenario $\langle X, \mathcal{M}, O \rangle$:

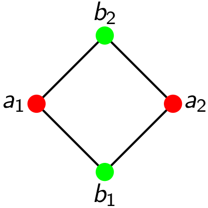
- X is a finite set of measurements or variables
- O is a finite set of outcomes or values
- $\mathcal{M}$ is a cover of X, indicating **joint measurability** (contexts)

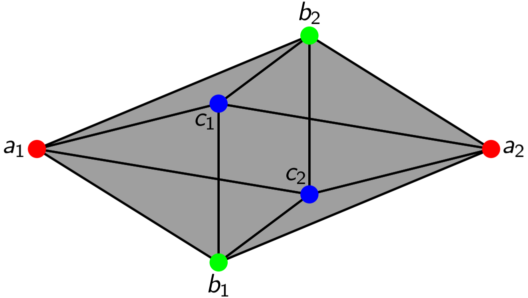
Example: (2,2,2) Bell scenario

- The set of variables is $X = \{a_1, a_2, b_1, b_2\}$.
- The outcomes are $O = \{0, 1\}$.
- The measurement contexts are:

$$\{\ \{a_1, b_1\}, \quad \{a_1, b_2\}, \quad \{a_2, b_1\}, \quad \{a_2, b_2\}\ \}.$$

Measurement scenarios





Examples: Bell-type scenarios, KS configurations, and more.

Another example: 18-vector Kochen–Specker

- A set of 18 variables, $X = \{A, \ldots, O\}$

Another example: 18-vector Kochen–Specker

- A set of 18 variables, $X = \{A, \ldots, O\}$
- A set of outcomes $O = \{0, 1\}$

Another example: 18-vector Kochen–Specker

- A set of 18 variables, $X = \{A, \ldots, O\}$
- A set of outcomes $O = \{0, 1\}$
- A measurement cover $\mathcal{M} = \{C_1, \ldots, C_9\}$, whose contexts C_i correspond to the columns in the following table:

U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9
A	A	H	H	B	I	P	P	Q
B	E	I	K	E	K	Q	R	R
C	F	C	G	M	N	D	F	M
D	G	J	L	N	O	J	L	O

Empirical Models

Joint outcome or **event** in a context C is $s \in O^C$, e.g.

$$s = [a_1 \mapsto 0, b_1 \mapsto 1] \ .$$

Empirical Models

Joint outcome or **event** in a context C is $s \in O^C$, e.g.

$$s = [a_1 \mapsto 0, b_1 \mapsto 1] \; .$$

Empirical model: family $\{e_C\}_{C \in \mathcal{M}}$ where $e_C \in \mathsf{Prob}(O^C)$ for $C \in \mathcal{M}$.

It specifies a probability distribution over the events in each context.
Each distribution is a row of the probability table.

Empirical Models

Joint outcome or **event** in a context C is $s \in O^C$, e.g.

$$s = [a_1 \mapsto 0, b_1 \mapsto 1] \ .$$

Empirical model: family $\{e_C\}_{C \in \mathcal{M}}$ where $e_C \in \mathrm{Prob}(O^C)$ for $C \in \mathcal{M}$.

It specifies a probability distribution over the events in each context.
Each distribution is a row of the probability table.

Compatibility condition: the distributions "agree on overlaps"

$$\forall\, C, C' \in \mathcal{M}. \quad e_C|_{C \cap C'} \ = \ e_{C'}|_{C \cap C'} \ .$$

Empirical Models

Joint outcome or **event** in a context C is $s \in O^C$, e.g.

$$s = [a_1 \mapsto 0, b_1 \mapsto 1] \ .$$

Empirical model: family $\{e_C\}_{C \in \mathcal{M}}$ where $e_C \in \mathrm{Prob}(O^C)$ for $C \in \mathcal{M}$.

It specifies a probability distribution over the events in each context.
Each distribution is a row of the probability table.

Compatibility condition: the distributions "agree on overlaps"

$$\forall\, C, C' \in \mathcal{M}. \quad e_C|_{C \cap C'} \ = \ e_{C'}|_{C \cap C'} \ .$$

In multipartite scenarios, compatibility = the **no-signalling** principle.

Contextuality

A (compatible) empirical model is **non-contextual** if there exists a **global distribution** $d \in \mathrm{Prob}(O^X)$ on the joint assignments of outcomes to all measurements that marginalises to all the e_C:

$$\exists\, d \in \mathrm{Prob}(O^X).\, \forall\, C \in \mathcal{M}. \quad d|_C \;=\; e_C \;.$$

Contextuality

A (compatible) empirical model is **non-contextual** if there exists a **global distribution** $d \in \mathrm{Prob}(O^X)$ on the joint assignments of outcomes to all measurements that marginalises to all the e_C:

$$\exists\, d \in \mathrm{Prob}(O^X).\ \forall\, C \in \mathcal{M}.\quad d|_C \ = \ e_C \ .$$

i.e. all the local information can be glued into a consistent global description.

Contextuality

A (compatible) empirical model is **non-contextual** if there exists a **global distribution** $d \in \text{Prob}(\mathcal{O}^X)$ on the joint assignments of outcomes to all measurements that marginalises to all the e_C:

$$\exists\, d \in \text{Prob}(\mathcal{O}^X).\ \forall\, C \in \mathcal{M}.\quad d|_C \ = \ e_C \ .$$

i.e. all the local information can be glued into a consistent global description.

Contextuality:
family of data which is **locally consistent** but **globally inconsistent**.

Contextuality

A (compatible) empirical model is **non-contextual** if there exists a **global distribution** $d \in \mathsf{Prob}(O^X)$ on the joint assignments of outcomes to all measurements that marginalises to all the e_C:

$$\exists\, d \in \mathsf{Prob}(O^X).\ \forall\, C \in \mathcal{M}.\quad d|_C \;=\; e_C\ .$$

i.e. all the local information can be glued into a consistent global description.

Contextuality:
family of data which is **locally consistent** but **globally inconsistent**.

The import of results such as Bell's and Bell–Kochen–Specker's theorems is that there are **contextual** empirical models arising from quantum mechanics.

Possibilistic collapse

- Given an empirical model e, define possibilistic model $\mathrm{poss}(e)$ by taking the support of each distributions.
- Contains the possibilistic, or logical, information of that model.

Possibilistic collapse

- Given an empirical model e, define possibilistic model $\mathsf{poss}(e)$ by taking the support of each distributions.
- Contains the possibilistic, or logical, information of that model.

	00	01	10	11
$a_1\,b_1$	$1/2$	0	0	$1/2$
$a_1\,b_2$	$3/8$	$1/8$	$1/8$	$3/8$
$a_2\,b_1$	$3/8$	$1/8$	$1/8$	$3/8$
$a_2\,b_2$	$1/8$	$3/8$	$3/8$	$1/8$

$\longmapsto$

	00	01	10	11
$a_1\,b_1$	1	0	0	1
$a_1\,b_2$	1	1	1	1
$a_2\,b_1$	1	1	1	1
$a_2\,b_2$	1	1	1	1

Possibilistic collapse

- Given an empirical model e, define possibilistic model $\mathsf{poss}(e)$ by taking the support of each distributions.
- Contains the possibilistic, or logical, information of that model.

	00	01	10	11
$a_1 b_1$		0	0	
$a_1 b_2$		$1/8$	$1/8$	
$a_2 b_1$		$1/8$	$1/8$	
$a_2 b_2$	$1/8$			$1/8$

$$\longmapsto$$

	00	01	10	11
$a_1 b_1$	1	0	0	1
$a_1 b_2$	1	1	1	1
$a_2 b_1$	1	1	1	1
$a_2 b_2$	1	1	1	1

In some instances, this is enough to witness contextuality!

Contextuality (topo)logically

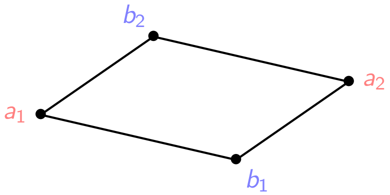
Hardy model

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

Contextuality (topo)logically

Hardy model

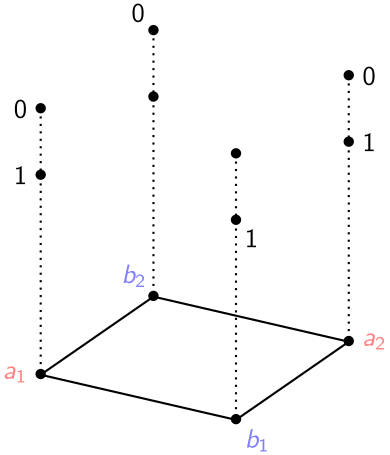
A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0



Contextuality (topo)logically

Hardy model

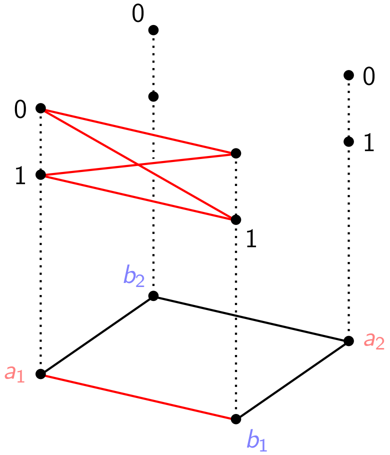
A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0



Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

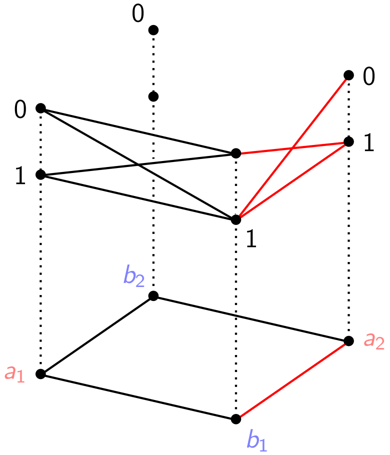


Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$

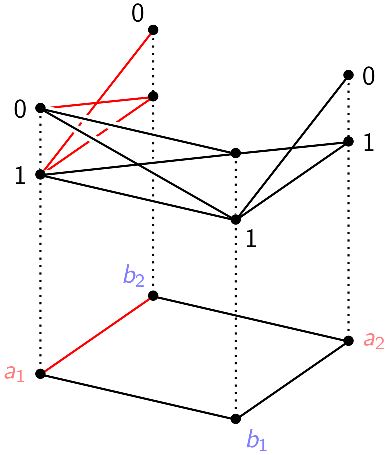


Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\quad$ $a_1 \vee b_2$

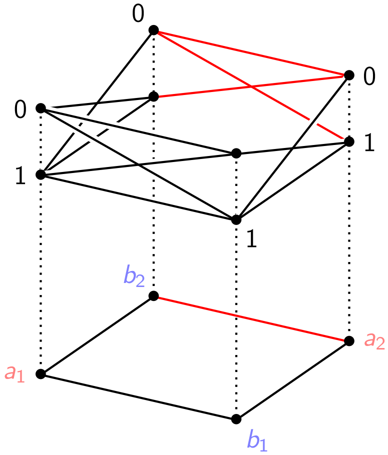


Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\quad$ $a_1 \vee b_2$ $\quad$ $\neg(a_2 \wedge b_2)$

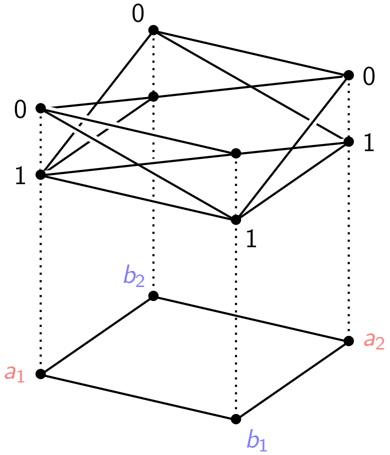


Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\quad a_1 \vee b_2$ $\quad \neg(a_2 \wedge b_2)$

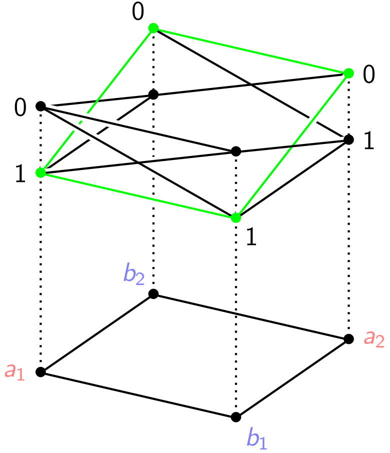


Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\quad$ $a_1 \vee b_2$ $\quad$ $\neg(a_2 \wedge b_2)$



There are some global sections,

Classical assignment: $[a_1 \mapsto 1, a_2 \mapsto 1, b_1 \mapsto 1, b_2 \mapsto 1]$

Contextuality (topo)logically

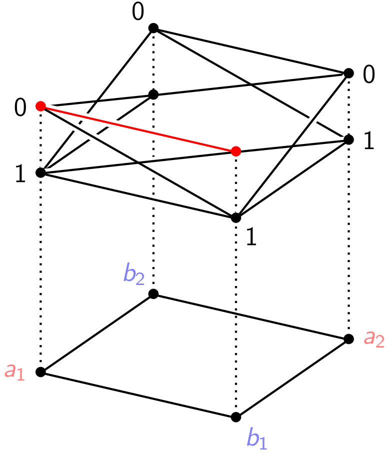
Hardy model

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad a_1 \vee b_2$ $\qquad \neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$

There are some global sections, but ...



Contextuality (topo)logically

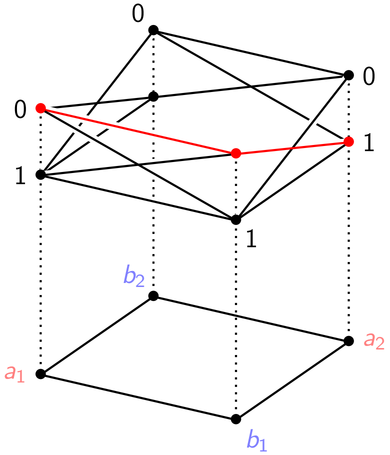
Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad$ $a_1 \vee b_2$ $\qquad$ $\neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$

There are some global sections, but . . .



Contextuality (topo)logically

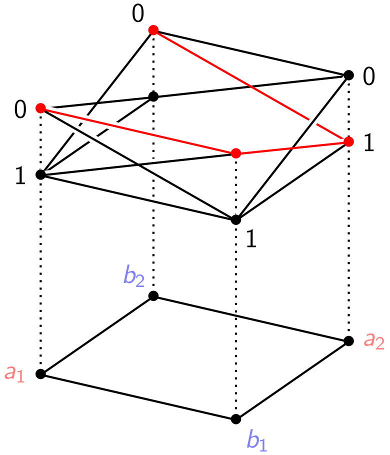
Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad$ $a_1 \vee b_2$ $\qquad$ $\neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$

There are some global sections, but . . .



Contextuality (topo)logically

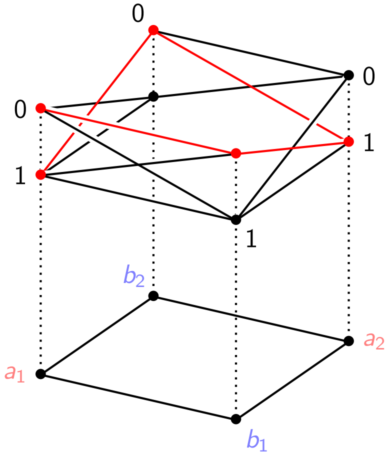
Hardy model

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad a_1 \vee b_2$ $\qquad \neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$

There are some global sections, but . . .



Contextuality (topo)logically

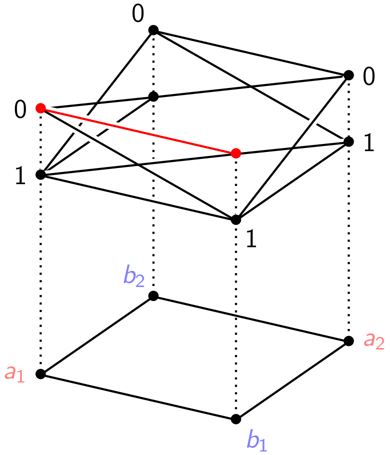
Hardy model

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad$ $a_1 \vee b_2$ $\qquad$ $\neg(a_2 \wedge b_2)$

$$[a_1 \mapsto 0, b_1 \mapsto 0]$$

There are some global sections, but . . .



Contextuality (topo)logically

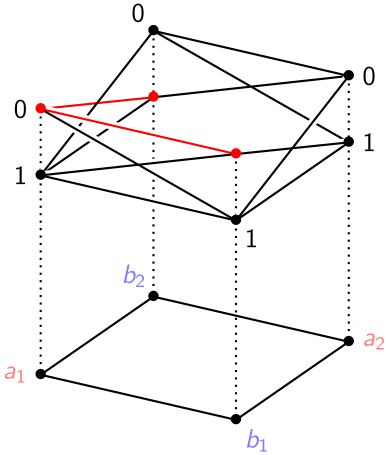
Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad$ $a_1 \vee b_2$ $\qquad$ $\neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$

There are some global sections, but . . .



Contextuality (topo)logically

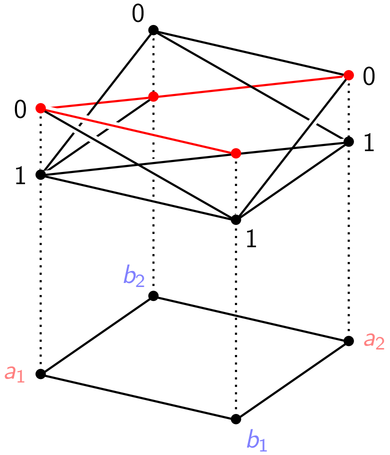
Hardy model

A	B	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad$ $a_1 \vee b_2$ $\qquad$ $\neg(a_2 \wedge b_2)$

$$[a_1 \mapsto 0, b_1 \mapsto 0]$$

There are some global sections, but . . .



Contextuality (topo)logically

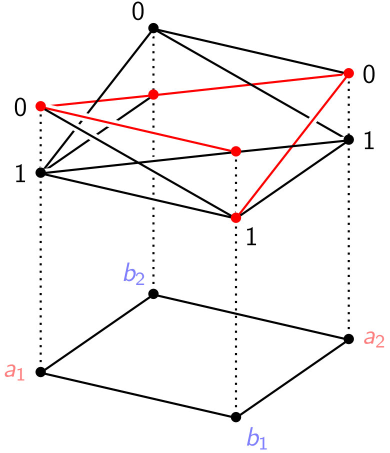
Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\quad$ $a_1 \vee b_2$ $\quad$ $\neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$

There are some global sections, but . . .



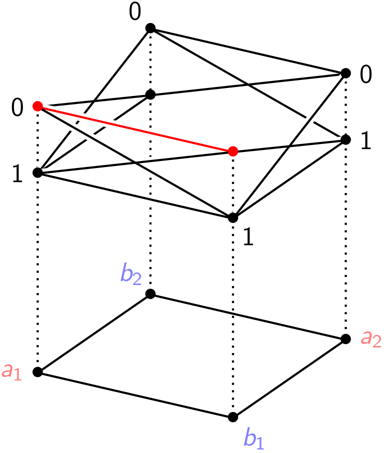
Contextuality (topo)logically

Hardy model

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	1	1	1
a_1	b_2	0	1	1	1
a_2	b_1	0	1	1	1
a_2	b_2	1	1	1	0

$a_2 \vee b_1$ $\qquad$ $a_1 \vee b_2$ $\qquad$ $\neg(a_2 \wedge b_2)$

$[a_1 \mapsto 0, b_1 \mapsto 0]$



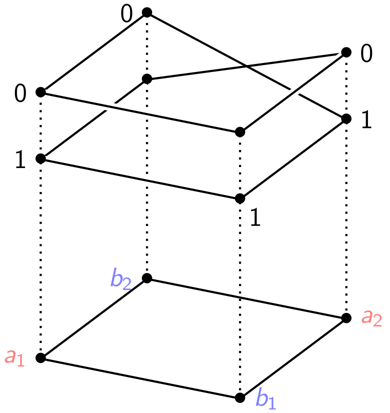
There are some global sections, but . . .

Logical contextuality: Not all sections extend to global ones.

Contextuality (topo)logically

Popescu–Rohrlich box

A	B	(0, 0)	(0, 1)	(1, 0)	(1, 1)
a_1	b_1	1	0	0	1
a_1	b_2	1	0	0	1
a_2	b_1	1	0	0	1
a_2	b_2	0	1	1	0



Strong contextuality:
no event can be extended to a global assignment.

$a_1 \leftrightarrow b_1 \quad a_1 \leftrightarrow b_2 \quad a_2 \leftrightarrow b_1 \quad a_2 \oplus b_2$

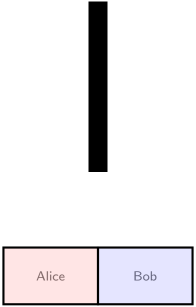
What does this have to do with quantum advantage?


IT'S NOT A
BUG
IT'S A
FEATURE

Non-local games

Alice and Bob cooperate in solving a task set by Verifier

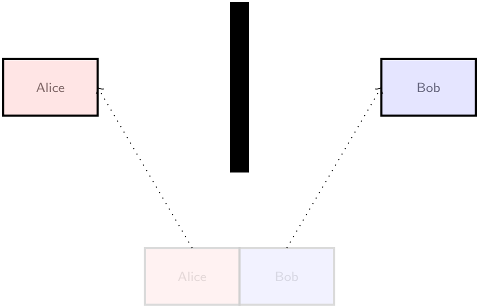
May share prior information,



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

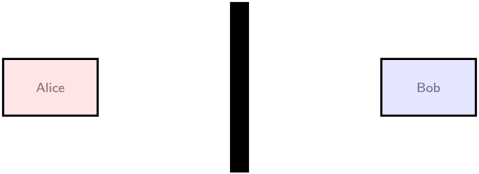
May share prior information,



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

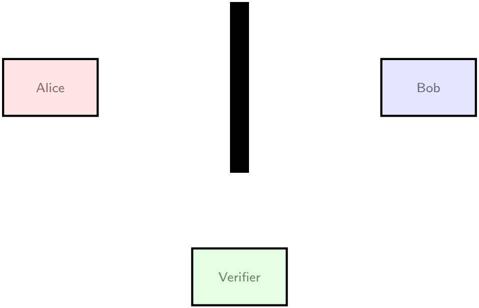
May share prior information, but cannot communicate once game starts



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

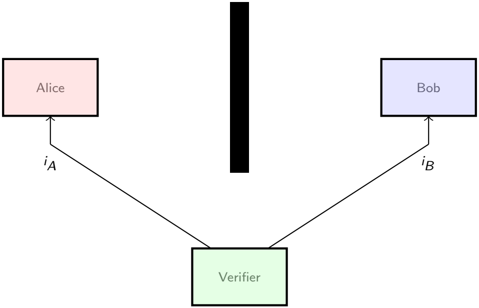
May share prior information, but cannot communicate once game starts



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

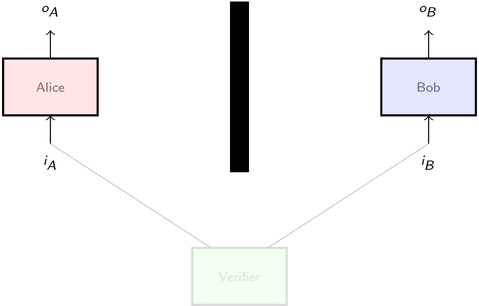
May share prior information, but cannot communicate once game starts



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

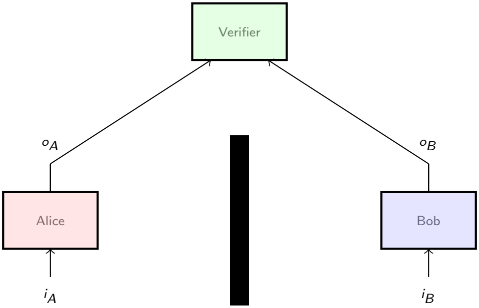
May share prior information, but cannot communicate once game starts



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

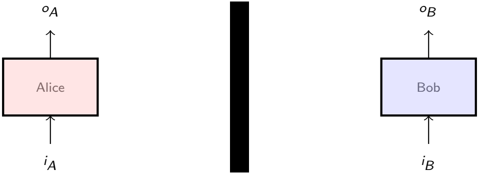
May share prior information, but cannot communicate once game starts



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

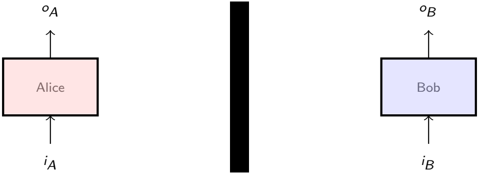
May share prior information, but cannot communicate once game starts



Non-local games

Alice and Bob cooperate in solving a task set by Verifier

May share prior information, but cannot communicate once game starts

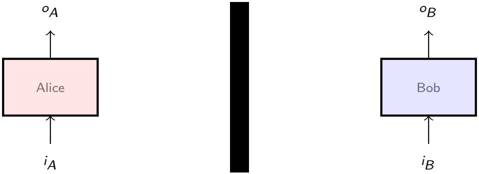


A strategy is described by the probabilities $P(\, o_A, o_B \mid i_A, i_B \,)$.

Non-local games

Alice and Bob cooperate in solving a task set by Verifier

May share prior information, but cannot communicate once game starts



A strategy is described by the probabilities $P(\, o_A, o_B \mid i_A, i_B \,)$.

A **perfect strategy** is one that wins with probability 1.

The AND game

- Verifier sends a bit to each of Alice and Bob, i_A and i_B.
- Each returns an output bit, o_A and o_B.
- Their outputs are combined by verifier: $o_A \oplus o_B$.
- They win if they implement the AND function:
  $o_A \oplus o_B = o_A \wedge o_B$

The AND game

- Verifier sends a bit to each of Alice and Bob, i_A and i_B.
- Each returns an output bit, o_A and o_B.
- Their outputs are combined by verifier: $o_A \oplus o_B$.
- They win if they implement the AND function:
  $o_A \oplus o_B = o_A \wedge o_B$

Classically, they can win with probablity at most $3/4$

Quantumly, the Bell table allows for a higher probability.
In fact, one can reach $(2 + \sqrt{2})/4 \approx 0.85$

Binary constraint systems games

Magic square:

- Fill with 0s and 1s
- rows and first two columns: even parity
- last column: odd parity

Binary constraint systems games

A	B	C
D	E	F
G	H	I

Magic square:

- Fill with 0s and 1s
- rows and first two columns: even parity
- last column: odd parity

System of linear equations over $\mathbb{Z}_2$:

$$A \oplus B \oplus C = 0 \qquad A \oplus D \oplus G = 0$$
$$D \oplus E \oplus F = 0 \qquad B \oplus E \oplus H = 0$$
$$G \oplus H \oplus I = 0 \qquad C \oplus F \oplus I = 1$$

Binary constraint systems games

A	B	C
D	E	F
G	H	I

Magic square:

- Fill with 0s and 1s
- rows and first two columns: even parity
- last column: odd parity

System of linear equations over $\mathbb{Z}_2$:

$$\begin{aligned} A \oplus B \oplus C &= 0 \\ D \oplus E \oplus F &= 0 \\ G \oplus H \oplus I &= 0 \end{aligned} \qquad \begin{aligned} A \oplus D \oplus G &= 0 \\ B \oplus E \oplus H &= 0 \\ C \oplus F \oplus I &= 1 \end{aligned}$$

Clearly, this is not satisfiable in $\mathbb{Z}_2$.

E.g.: Binary contraint satisfaction game

- Verifier sends an **equation** to Alice
- and a **variable** to Bob

E.g.: Binary contraint satisfaction game

- Verifier sends an **equation** to Alice
- and a **variable** to Bob

- Alice returns an **assignment** for the variables in her equation
- Bob returns a **value** for his variable

E.g.: Binary contraint satisfaction game

- Verifier sends an **equation** to Alice
- and a **variable** to Bob

- Alice returns an **assignment** for the variables in her equation
- Bob returns a **value** for his variable

- They win the play if:
  - Alice's assignment satisfies the equation
  - Bob's value is consistent with Alice's assigment

E.g.: Binary contraint satisfaction game

- Verifier sends an **equation** to Alice
- and a **variable** to Bob

- Alice returns an **assignment** for the variables in her equation
- Bob returns a **value** for his variable

- They win the play if:
  - Alice's assignment satisfies the equation
  - Bob's value is consistent with Alice's assigment

Classically, Alice and Bob have a perfect strategy if and only if there is an assignment to all variables satisfying the system of equations.

E.g.: Binary contraint satisfaction game

- Verifier sends an **equation** to Alice
- and a **variable** to Bob

- Alice returns an **assignment** for the variables in her equation
- Bob returns a **value** for his variable

- They win the play if:
  - Alice's assignment satisfies the equation
  - Bob's value is consistent with Alice's assigment

Classically, Alice and Bob have a perfect strategy if and only if there is an assignment to all variables satisfying the system of equations.

But using quantum resources, they can win the Magic Square game with probability 1, using Mermin's construction.

E.g.: Binary contraint satisfaction game

- Verifier sends an **equation** to Alice
- and a **variable** to Bob

- Alice returns an **assignment** for the variables in her equation
- Bob returns a **value** for his variable

- They win the play if:
  - Alice's assignment satisfies the equation
  - Bob's value is consistent with Alice's assigment

Classically, Alice and Bob have a perfect strategy if and only if there is an assignment to all variables satisfying the system of equations.

But using quantum resources, they can win the Magic Square game with probability 1, using Mermin's construction.

The system has a **quantum solution** but no classical solution!

Contextual fraction and quantum advantages

- Contextuality has been associated with quantum advantage in information-processing and computational tasks.

Contextual fraction and quantum advantages

- Contextuality has been associated with quantum advantage in information-processing and computational tasks.
  - **Non-local games**
    XOR games (CHSH; Cleve–Høyer–Toner–Watrous)
    quantum graph homomorphisms (Mančinska–Roberson)
    constraint satisfaction (Cleve–Mittal)
    etc. (Abramsky–B–de Silva–Zapata)

Contextual fraction and quantum advantages

- Contextuality has been associated with quantum advantage in information-processing and computational tasks.
  - **Non-local games**
    XOR games (CHSH; Cleve–Høyer–Toner–Watrous)
    quantum graph homomorphisms (Mančinska–Roberson)
    constraint satisfaction (Cleve–Mittal)
    etc. (Abramsky–B–de Silva–Zapata)
  - **MBQC**
    Raussendorf (2013)
    "Contextuality in measurement-based quantum computation"

Contextual fraction and quantum advantages

- Contextuality has been associated with quantum advantage in information-processing and computational tasks.
  - **Non-local games**
    XOR games (CHSH; Cleve–Høyer–Toner–Watrous)
    quantum graph homomorphisms (Mančinska–Roberson)
    constraint satisfaction (Cleve–Mittal)
    etc. (Abramsky–B–de Silva–Zapata)
  - **MBQC**
    Raussendorf (2013)
    "Contextuality in measurement-based quantum computation"
  - **MSD**
    Howard–Wallman–Veith–Emerson (2014)
    "Contextuality supplies the 'magic' for quantum computation"

Contextual fraction and quantum advantages

- Contextuality has been associated with quantum advantage in information-processing and computational tasks.
  - **Non-local games**
    XOR games (CHSH; Cleve–Høyer–Toner–Watrous)
    quantum graph homomorphisms (Mančinska–Roberson)
    constraint satisfaction (Cleve–Mittal)
    etc. (Abramsky–B–de Silva–Zapata)
  - **MBQC**
    Raussendorf (2013)
    "Contextuality in measurement-based quantum computation"
  - **MSD**
    Howard–Wallman–Veith–Emerson (2014)
    "Contextuality supplies the 'magic' for quantum computation"

- Measure of contextuality $\rightsquigarrow$ quantify such advantages.

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

- **General**, i.e. applicable to any measurement scenario

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

- **General**, i.e. applicable to any measurement scenario
- **Normalised**, allowing comparison across scenarios
  0 for non-contextuality . . . 1 for strong contextuality

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

- **General**, i.e. applicable to any measurement scenario
- **Normalised**, allowing comparison across scenarios
  0 for non-contextuality . . . 1 for strong contextuality
- Computable using **linear programming**

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

- **General**, i.e. applicable to any measurement scenario
- **Normalised**, allowing comparison across scenarios
  0 for non-contextuality . . . 1 for strong contextuality
- Computable using **linear programming**
- Precise relationship to **violations of Bell inequalities** (Dual LP)

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

- **General**, i.e. applicable to any measurement scenario
- **Normalised**, allowing comparison across scenarios
  0 for non-contextuality . . . 1 for strong contextuality
- Computable using **linear programming**
- Precise relationship to **violations of Bell inequalities** (Dual LP)
- **Monotone** wrt operations that don't introduce contextuality
  $\rightsquigarrow$ **resource theory**

Measuring contextuality

We introduce the **contextual fraction**
(generalising the notion of non-local fraction)

It satisfies a number of desirable properties:

- **General**, i.e. applicable to any measurement scenario
- **Normalised**, allowing comparison across scenarios
  0 for non-contextuality . . . 1 for strong contextuality
- Computable using **linear programming**
- Precise relationship to **violations of Bell inequalities** (Dual LP)
- **Monotone** wrt operations that don't introduce contextuality
  $\rightsquigarrow$ **resource theory**
- Relates to quantifiable **advantages** in QC and QIP tasks

The contextual fraction

Non-contextuality: global distribution $d \in \mathrm{Prob}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}} .\ d|_C \;=\; e_C \ .$$

The contextual fraction

Non-contextuality: global distribution $d \in \mathrm{Prob}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}} .\ d|_C \;=\; e_C \;.$$

Which fraction of a model admits a non-contextual explanation?

The contextual fraction

Non-contextuality: global distribution $d \in \mathrm{Prob}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ d|_C \ = \ e_C \ .$$

Which fraction of a model admits a non-contextual explanation?

Consider **subdistributions** $c \in \mathrm{SubProb}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ c|_C \ \leq \ e_C \ .$$

Non-contextual fraction: maximum weight of such a subdistribution.

The contextual fraction

Non-contextuality: global distribution $d \in \mathsf{Prob}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ d|_C \ = \ e_C \ .$$

Which fraction of a model admits a non-contextual explanation?

Consider **subdistributions** $c \in \mathsf{SubProb}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ c|_C \ \leq \ e_C \ .$$

Non-contextual fraction: maximum weight of such a subdistribution.

Equivalently, maximum weight λ over all convex decompositions

$$e = \lambda e^{NC} + (1 - \lambda) e'$$

where e^{NC} is a non-contextual model.

The contextual fraction

Non-contextuality: global distribution $d \in \mathsf{Prob}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ d|_C \ = \ e_C \ .$$

Which fraction of a model admits a non-contextual explanation?

Consider **subdistributions** $c \in \mathsf{SubProb}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ c|_C \ \leq \ e_C \ .$$

Non-contextual fraction: maximum weight of such a subdistribution.

Equivalently, maximum weight λ over all convex decompositions

$$e = \lambda e^{NC} + (1 - \lambda) e'$$

where e^{NC} is a non-contextual model.

The contextual fraction

Non-contextuality: global distribution $d \in \mathsf{Prob}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ d|_C \ = \ e_C \ .$$

Which fraction of a model admits a non-contextual explanation?

Consider **subdistributions** $c \in \mathsf{SubProb}(O^X)$ such that:

$$\forall_{C \in \mathcal{M}}.\ c|_C \ \leq \ e_C \ .$$

Non-contextual fraction: maximum weight of such a subdistribution.

Equivalently, maximum weight λ over all convex decompositions

$$e = \lambda e^{NC} + (1 - \lambda) e^{SC}$$

where e^{NC} is a non-contextual model. e^{SC} is strongly contextual!

$$\mathsf{NCF}(e) = \lambda \qquad \mathsf{CF}(e) = 1 - \lambda$$

Contextuality and MBQC

E.g. Raussendorf (2013) ℓ2-MBQC

Contextuality and MBQC

E.g. Raussendorf (2013) $\ell 2$-MBQC

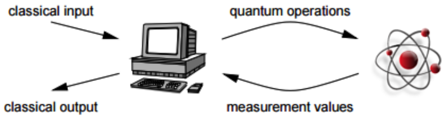
- measurement-based quantum computing scheme (m input bits, l output bits, n parties)

Contextuality and MBQC

E.g. Raussendorf (2013) ℓ2-MBQC

- measurement-based quantum computing scheme (m input bits, l output bits, n parties)
- classical control:
  - pre-processes input
  - determines the flow of measurements
  - post-processes to produce the output

  only $\mathbb{Z}_2$-linear computations.



Contextuality and MBQC

Contextuality and MBQC

- Additional power to compute non-linear functions resides in using resources displaying contextual correlations.

$$\oplus \mathbf{L} \longrightarrow \mathbf{P}$$

Contextuality and MBQC

- Additional power to compute non-linear functions resides in using resources displaying contextual correlations.

$$\oplus\mathbf{L} \longrightarrow \mathbf{P}$$

- Raussendorf (2013): If an $\ell 2$-MBQC **deterministically** computes a non-linear Boolean function $f : 2^m \longrightarrow 2^l$ then the resource must be **strongly contextual**.

Contextuality and MBQC

- Additional power to compute non-linear functions resides in using resources displaying contextual correlations.

$$\oplus\mathbf{L} \longrightarrow \mathbf{P}$$

- Raussendorf (2013): If an $\ell 2$-MBQC **deterministically** computes a non-linear Boolean function $f : 2^m \longrightarrow 2^l$ then the resource must be **strongly contextual**.

- Probabilistic version: non-linear function computed with sufficently large probability of success implies contextuality.

Contextual fraction and MBQC

- **Goal**: Compute Boolean function $f : 2^m \longrightarrow 2^l$ using ℓ2-MBQC

Contextual fraction and MBQC

- **Goal**: Compute Boolean function $f : 2^m \longrightarrow 2^l$ using ℓ2-MBQC
- **Hardness of the problem**

$$\nu(f) := \min \{ d(f, g) \mid g \text{ is } \mathbb{Z}_2\text{-linear} \}$$

(average distance between f and closest $\mathbb{Z}_2$-linear function)

where for Boolean functions f and g, $d(f, g) := 2^{-m} |\{ \mathbf{i} \in 2^m \mid f(\mathbf{i}) \neq g(\mathbf{i}) \}|$.

Contextual fraction and MBQC

- **Goal**: Compute Boolean function $f : 2^m \longrightarrow 2^l$ using ℓ2-MBQC

- **Hardness of the problem**

$$\nu(f) := \min \{ d(f, g) \mid g \text{ is } \mathbb{Z}_2\text{-linear} \}$$

(average distance between f and closest $\mathbb{Z}_2$-linear function)

where for Boolean functions f and g, $d(f, g) := 2^{-m} |\{ \mathbf{i} \in 2^m \mid f(\mathbf{i}) \neq g(\mathbf{i}) \}$.

- **Average probability of success** computing f (over all 2^m possible inputs): $\bar{p}_S$.

Contextual fraction and MBQC

- **Goal**: Compute Boolean function $f : 2^m \longrightarrow 2^l$ using ℓ2-MBQC
- **Hardness of the problem**

$$\nu(f) := \min \{ d(f,g) \mid g \text{ is } \mathbb{Z}_2\text{-linear} \}$$

(average distance between f and closest $\mathbb{Z}_2$-linear function)

where for Boolean functions f and g, $d(f,g) := 2^{-m} |\{\mathbf{i} \in 2^m \mid f(\mathbf{i}) \neq g(\mathbf{i})\}$.

- **Average probability of success** computing f (over all 2^m possible inputs): $\bar{p}_S$.
- Then,

$$\boxed{1 - \bar{p}_S \quad \geq \quad \mathsf{NCF}(e)\, \nu(f)}$$

Contextuality as a resource: Algebra of empirical models

- Think of empirical models as black boxes

Contextuality as a resource: Algebra of empirical models

- Think of empirical models as black boxes
- What operations can we perform (*non-contextually*) on them?

Contextuality as a resource: Algebra of empirical models

- Think of empirical models as black boxes
- What operations can we perform (*non-contextually*) on them?
- We write type statements

$$e : \langle X, \mathcal{M}, O \rangle$$

to mean that e is a (compatible) emprical model on $\langle X, \mathcal{M}, O \rangle$.

Contextuality as a resource: Algebra of empirical models

- Think of empirical models as black boxes
- What operations can we perform (*non-contextually*) on them?
- We write type statements

$$e : \langle X, \mathcal{M}, O \rangle$$

  to mean that e is a (compatible) emprical model on $\langle X, \mathcal{M}, O \rangle$.

- The operations remind one of process algebras.

Operations and the contextual fraction

Operations and the contextual fraction

Relabelling $e[\alpha]$

Operations and the contextual fraction

Relabelling $e[\alpha]$

Restriction $e \restriction \mathcal{M}'$

Operations and the contextual fraction

Relabelling $e[\alpha]$

Restriction $e \upharpoonright \mathcal{M}'$

Coarse-graining e/f

Operations and the contextual fraction

Relabelling	$e[\alpha]$
Restriction	$e \restriction \mathcal{M}'$
Coarse-graining	e/f
Mixing	$\lambda e + (1-\lambda)e'$

Operations and the contextual fraction

Relabelling	$e[\alpha]$
Restriction	$e \upharpoonright \mathcal{M}'$
Coarse-graining	e/f
Mixing	$\lambda e + (1 - \lambda)e'$
Choice	$e \,\&\, e'$

Operations and the contextual fraction

Relabelling	$e[\alpha]$
Restriction	$e \upharpoonright \mathcal{M}'$
Coarse-graining	e/f
Mixing	$\lambda e + (1 - \lambda)e'$
Choice	$e \,\&\, e'$
Tensor	$e_1 \otimes e_2$

Operations and the contextual fraction

Relabelling	$e[\alpha]$
Restriction	$e \upharpoonright \mathcal{M}'$
Coarse-graining	e/f
Mixing	$\lambda e + (1-\lambda)e'$
Choice	$e \,\&\, e'$
Tensor	$e_1 \otimes e_2$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$e \restriction \mathcal{M}'$
Coarse-graining	e/f
Mixing	$\lambda e + (1 - \lambda) e'$
Choice	$e \,\&\, e'$
Tensor	$e_1 \otimes e_2$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$\mathsf{CF}(e \restriction \mathcal{M}') \leq \mathsf{CF}(e)$
Coarse-graining	e/f
Mixing	$\lambda e + (1-\lambda)e'$
Choice	$e \,\&\, e'$
Tensor	$e_1 \otimes e_2$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$\mathsf{CF}(e \restriction \mathcal{M}') \leq \mathsf{CF}(e)$
Coarse-graining	$\mathsf{CF}(e/f) \leq CF(e)$
Mixing	$\lambda e + (1-\lambda)e'$
Choice	$e \,\&\, e'$
Tensor	$e_1 \otimes e_2$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$\mathsf{CF}(e \restriction \mathcal{M}') \leq \mathsf{CF}(e)$
Coarse-graining	$\mathsf{CF}(e/f) \leq CF(e)$
Mixing	$CF(\lambda e + (1-\lambda)e') \leq \lambda \mathsf{CF}(e) + (1-\lambda)\mathsf{CF}(e')$
Choice	$e \,\&\, e'$
Tensor	$e_1 \otimes e_2$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$\mathsf{CF}(e \restriction \mathcal{M}') \leq \mathsf{CF}(e)$
Coarse-graining	$\mathsf{CF}(e/f) \leq CF(e)$
Mixing	$CF(\lambda e + (1-\lambda)e') \leq \lambda \mathsf{CF}(e) + (1-\lambda)\mathsf{CF}(e')$
Choice	$\mathsf{CF}(e \,\&\, e') = \max\{\mathsf{CF}(e), \mathsf{CF}(e')\}$
Tensor	$e_1 \otimes e_2$ $\mathsf{NCF}(e_1 \otimes e_2) = \mathsf{NCF}(e_1)\,\mathsf{NCF}(e_2)$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$\mathsf{CF}(e \restriction \mathcal{M}') \leq \mathsf{CF}(e)$
Coarse-graining	$\mathsf{CF}(e/f) \leq CF(e)$
Mixing	$CF(\lambda e + (1-\lambda)e') \leq \lambda \mathsf{CF}(e) + (1-\lambda)\mathsf{CF}(e')$
Choice	$\mathsf{CF}(e \,\&\, e') = \max\{\mathsf{CF}(e), \mathsf{CF}(e')\}$
Tensor	$\mathsf{CF}(e_1 \otimes e_2) = \mathsf{CF}(e_1) + \mathsf{CF}(e_2) - \mathsf{CF}(e_1)\mathsf{CF}(e_2)$ $\mathsf{NCF}(e_1 \otimes e_2) = \mathsf{NCF}(e_1)\,\mathsf{NCF}(e_2)$
Sequencing	$e_1 ; e_2$

Operations and the contextual fraction

Relabelling	$\mathsf{CF}(e[\alpha]) = \mathsf{CF}(e)$
Restriction	$\mathsf{CF}(e \restriction \mathcal{M}') \leq \mathsf{CF}(e)$
Coarse-graining	$\mathsf{CF}(e/f) \leq CF(e)$
Mixing	$CF(\lambda e + (1-\lambda)e') \leq \lambda \mathsf{CF}(e) + (1-\lambda)\mathsf{CF}(e')$
Choice	$\mathsf{CF}(e \,\&\, e') = \max\{\mathsf{CF}(e), \mathsf{CF}(e')\}$
Tensor	$\mathsf{CF}(e_1 \otimes e_2) = \mathsf{CF}(e_1) + \mathsf{CF}(e_2) - \mathsf{CF}(e_1)\mathsf{CF}(e_2)$
	$\mathsf{NCF}(e_1 \otimes e_2) = \mathsf{NCF}(e_1)\,\mathsf{NCF}(e_2)$
Sequencing	$\mathsf{CF}(e_1 \otimes e_2) \leq \mathsf{CF}(e_1) + \mathsf{CF}(e_2) - \mathsf{CF}(e_1)\mathsf{CF}(e_2)$
	$\mathsf{NCF}(e_1 ; e_2) \geq \mathsf{NCF}(e_1)\,\mathsf{NCF}(e_2)$

Resource theory of contextuality

(some work in progress)

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

- Sequencing:

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

- Sequencing:
  - so far, it hides middle steps

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

- Sequencing:
  - so far, it hides middle steps
  - not doing so leads to notion of causal empirical models.

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

- Sequencing:
  - so far, it hides middle steps
  - not doing so leads to notion of causal empirical models.

- Allow natural expression of measurement-based computation with feed-forward, in a device-independent form:

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

- Sequencing:
  - so far, it hides middle steps
  - not doing so leads to notion of causal empirical models.

- Allow natural expression of measurement-based computation with feed-forward, in a device-independent form:
  - One can measure a non-maximal context (face σ of complex)

Resource theory of contextuality

(some work in progress)

- Resource theory *a la* Coecke–Fritz–Spekkens.
  (resource theory of combinable processes)

- Device-independent processes
  - Operations remind one of process algebra
  - Process calculus:
    operational semantics by (probabilistic) transitions
  - bissimulation, metric / approximation
  - (modal) logic for device-independent processes

- Sequencing:
  - so far, it hides middle steps
  - not doing so leads to notion of causal empirical models.

- Allow natural expression of measurement-based computation with feed-forward, in a device-independent form:
  - One can measure a non-maximal context (face σ of complex)
  - leaving a model on scenario $\mathsf{lk}_\sigma \mathcal{M}$

Questions...

?