Computational-topological approach to the classification of global dynamics of multi-parameter systems

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July 24, 2008

Abstract

We introduce an algorithmic method for obtaining a database of global dynamical behaviours encountered in a multi-parameter family of discrete dynamical systems on a bounded set in $\mathbf{R}^n$. In this approach, the region of parameters and the phase space are both divided into a finite number of boxes, and for each parameter box the corresponding family of dynamical systems is represented by means of a combinatorial multivalued map. This map is an outer approximation valid for the entire family and is computed with interval arithmetic. The analysis of dynamics is conducted at the combinatorial level with fast graph algorithms. The recurrent dynamics is captured in a combinatorial version of the Conley-Morse decomposition, leaving gradient-like dynamics in the remainder of the phase space, which together provide a schematic picture of global dynamics. Automatic homology computation algorithms are used to compute the Conley index which allows to reconstruct certain properties of invariant sets found in the combinatorial way. Morse decompositions for adjacent parameter boxes are matched and provide rigorous continuation results, and also allow to detect possible bifurcations. A nonlinear overcompensatory Leslie population model is used as a sample dynamical system which illustrates the effectiveness of this approach.

This is joint work with Zin Arai (Kyoto University), William Kalies (Florida Atlantic University), Hiroshi Kokubu (Kyoto University), Konstantin Mischaikow (Rutgers University), and Hiroe Oka (Ryukoku University).

AMS Subject Classification. Primary: 37B35. Secondary: 37B30, 37M99, 37N25, 92-08.

Keywords: Dynamical system; Global dynamics; Conley index; Morse decomposition; Leslie population models; Combinatorial dynamics; Rigorous numerics; Computational dynamics; Multiparameter system