Mauricio Peixoto

Academic Career

1953	together with Leopoldo Nachbin helped to found IMPA (Instituto de Nacional de Matematica Pura e Aplicada), to which has been associated ever since and is a Emeritus Researcher at the present;
1957-1958	visited Princeton and met S. Lefschetz and later S. Smale who in 1960 spent six months at IMPA and did there pioneering work of fundamental importance;
1964-1968	Professor at Brown University, Providence, R.I., U.S.A.;
1972-1978	Professor at University of Sao Paulo, Brazil.

Awards and Honors

1969	Moinho Santista Prize for Mathematics;
1975-1977	President of the Brazilian Mathematical Society;
1979-1980	President of the National Research Council;
1981-1991	President of the Brazilian Academy of Sciences;
1987	received the Third World Academy of Science Award in Mathematics;

Main Research Achievements

Roughly speaking most of his work his concerned with the global theory of ordinary differential equations. The best known part of this work corresponds to the papers [15], [17], [19] and this is nowadays referred to as Peixoto's Theorem. In [40] Peixoto found find a careful presentation of how it came about and how it was instrumental in putting the qualitative theory of flows on differentiable manifolds on a solid set-theoretical basis with reasonably well defined goals and problems exhibiting a certain unity.

The gist of his contribution here is: (i) the introduction of the space of all flows; (ii) the modification of the original definition of structural stability by Andronov - Pontrjagin freeing it from the requirement of a small, ? - homeomorphism; (iii) the recognition of the importance and of the difficulty of the differentiable “closing lemma". Concerning (ii) it should be remarked that this modified non ϵ - definition of structural stability was introduced in 1959, [15, p. 201] and is nowadays the usual definition of structural stability. We remark that as late as 1986, Anosov in a survey article about structural stability [Structurally stable systems, Proc. Steklov Inst. Math., issue 4, pp. 61-95] still refers to this usual definition as “structural stability in the sense of Peixoto".

The above Theorem was the starting point for the setting up of a high dimension qualitative theory of flows and diffeomorphisms on manifolds that was undertaken by Smale and his school in the sixties and seventies and continues to this day.

If we add to this the remarkable contributions of Kolmogorov, Arnold, Moser and others who look at these problems from a somewhat different, more metric point of view we get a vast body of knowledge that constitutes what is called nowadays Dynamical Systems. Thanks to the immense progress of computation techniques these theoretical concepts became more and more amenable to applications in the physical sciences. This seems to be the reason why the above Theorem is finding its place in many text books of applied mathematics even at the undergraduate level. A final comment about the above Theorem is that a natural complement to it is found in [28] where it is given a complete classification of structurally stable flows (i. e. Morse-Smale) on compact surfaces. This is done by means of “distinguished graphs"associated to such flows. In a recent paper by X. Wang [Ergod. Th. Dynam. Sys. (1990), 10, 565-597] a close relationship is shown to exist between these graphs and the C^*- algebra of the corresponding flows. This approach offers a kind of algebraic substratum to the distinguished graphs of [28] and ties up nicely the above classification with modern algebraic trends. We now turn to another aspect of Peixoto’s list of Publications. I wish to point out to a string of some 12 papers starting at my very first contribution [1] and eleven of the last ones ([34] - [36], [39], [40] and [42] - [47]). They are all somehow connected with the 2-point boundary value problem for a second order ordinary differential equation and more precisely to the problem of counting how many solutions do pass through the end points. In the case where only one such solution exists the subject relates naturally to the concept of generalized convexity with respect to the family of solutions of the equation $y'' = F(x, y, y')$. In particular I proved the following characterization theorem [7]: a function g(x) is convex with respect to the family of solutions of the above equation if and only if $g'' > F(x, g, g')$. This theorem generalizes the classical result that $g''(x) > 0$ is a necessary and sufficient condition for ordinary convexity of g. Ordinary convexity ammounts to generalized convexity with respect to the solutions of the equation $y''(x) = 0$. An application of our theorem to a mechanical problem was made in [11, pp. 102-108]. Giving up the very special case where the 2-point problem has always a unique solution, Peixoto came back to this problem in [25] with the knowledge I had acquired in dynamical system theory and put some genericity into the picture. So [25] is some kind of Kupka - Smale theorem (Kupka thesis at IMPA) in the context of the 2-point problem. What takes the places of the stable and unstable manifolds of the K-S theorem are the “lifted manifolds"at each point i.e. at each point Peixoto makes a blow up in 3-space of the totality of the trajectories through the point. We now come to [34] where Peixoto introduced the concept of focal decomposition (originally called sigma-decomposition) associated to the 2-point problem. Given a second order equation $x'' = F(t, x, x^{'})$ and fixing a point $A_0(t_0, x_0)$, each other point (t, x) is labeled by an integer i, the number of solutions i of the equation through (t0, x0) and (t,x). We then call ?i the totallity of points (t,x) to which the index i has been assigned. The fundamental problem then is: to study the nature of the sets σ_i and of the decomposition of the plane determined by them. In [35,36], joint work with R. Thom Peixoto generalized the above problem letting the base point vary also so that we get a sigma decomposition of R^4 into sets Σ_i . They then show the existence of a certain 4-dimensional manifold $\Omega \subset R^6$ and of a projection $\Pi : R^6 \to R^4$ such that $(t_1, x_1, t_2, x_2) \in \Sigma_i$ if and only if the cardinality of $(\Pi|\Omega)^{-1}(t_1, x_1, t_2, x_2)$ is i. From this and from results of Hironaka and Thom, it follows that when the differential equation is analytic and the projection $(\Pi|\Omega)$ is proper, then calling δ the diagonal $t_1 = t_2$ in $R_4(t_1, x_1, t_2, x_2)$ we have that there is a Whitney stratification of $R^4 - \delta$ such that each $\Sigma_i - \delta$ is the union of strata. In [35] we construct the focal decomposition associated to the pendulum equation $x^{''} + sinx = 0$. It exhibits non empty σ_i for all indices i. In [38], in collaboration with A. R. Silva, Peixoto showed that some results of S. Bernstein fit nicely with the results of [34, 35]. In [39], joint work with Kupka, Peixoto extended focal decomposition to the case of geodesics. In the case of the flat torus the corresponding focal decomposition, Fig. 1 of [39], is a most fascinating object, identical with the extension of the equation $x_2 + y_2 = N$ to the whole plane (in a natural sense) and to the Brillouin zones of a cubic crystal.

Spontaneous commentarys from other authors

Steve Smale, in the book “The Mathematics of Time"(Springer Verlag 1980), selects six of his papers on Dynamical Systems and Economy and among them, the article "What is global analysis?"(Am. Math. Monthly vol. 76 ,1969, pp.4-9) is essentially PeixotoÕs theorem. In the same book he gives the following testimony: “It was around 1958 that I first met Mauricio Peixoto. We were introduced by Lima who was finishing his Ph.D. at that time with Ed Spanier. Through Lefschetz, Peixoto had become interested in structural stability and he showed me his own results on structural stability on the disk D2 (in a paper that was to appear in the Annals of Mathematics, 1959). I was immediately enthusiastic, not only about what he was doing but with the possibility that, using my topology background, I could extend his work to n dimensions. “Peixoto told me that he had met Pontryagin, who said that he did not believe in structural stability in dimensions greater than two, but that only increased the challenge."René Thom in the article “The role of qualitative dynamics in applied sciences"("Geometric dynamics", edited by Jacob Palis, Lecture Notes in Mathematics, number 1007, Springer Verlag, 1983, pp. 784-788), wrote:

“Now the global theory of topological stability of flows, originated by Poincaré, and developed by him for the study of the 3 - body problem (discovery of homoclinic, heteroclinic points) found its first major development with G.D. Birkhoff (1920), who introduced the fundamental notions of wandering, and non-wandering points. The second decisive progress came from the Soviet School, when Andronov-Pontrjagin, introduced the notion of structural stability of flows (1930). The third decisive progress came with the results of S. Smale and M.M. Peixoto, e.g. the density of stable flows on surfaces."

List of publications

1. Sobre las soluciones de la ecuación $yy" = \phi(y')$ que pasan por dos puntos del semi-plano y > 0. Revista de la Union Matemática Argentina, Vol. XI, 1946, p. 84-91.

2. Sistemas nŃo holônomos; 68 pages, Rio de Janeiro, 1947.

3. Princípios variacionais de Hamilton e da menor acão; 55 pages, Rio de Janeiro, 1947.

4. Uma desigualdade entre números positivos. Gazeta de Matemática, Vol. 9, 1948,

p. 19-20.

5. On the existence of derivative of generalized convex functions. Summa Brasiliensis Mathematicae, Vol. 2, 1948, p. 35-42.

6. Convexidade das curvas. 66 pages. Notas de Matemática number 6, Rio de Janeiro, 1948.

7. Generalized convex functions and second order differential inequalities. Bulletin of the American Mathematical Society, Vol. 55, number 6, 1949, p. 563-572.

8. On convexity. Anais da Academia Brasileira de Ciências, Vol. XXI, 1949, p. 291-302.

9. Note on uniform continuity. Proceedings of the International Congress of Mathematicians, Vol. 1, 1950, p. 385. (in collaboration with A.A. Monteiro).

Le Nombre de Lebesgue et la continuitŐ uniforme. Portugaliae Mathematica, Vol. 10, 1951, p. 105-113. (in collaboration with A.A. Monteiro).

11. Equacões gerais da dinL'mica; 110 pages, Rio de Janeiro,1951.

12. Note on structurally stable systems. Anais da Academia Brasileira de Ciências, Vol. 27, 1955, p. 35.

13. On integral invariants. Anais da Academia Brasileira de Ciências, Vol. 38, 1956, p. XXV.

14. On structural stability. International Congress of Mathematicians, Edinburgh, 1958. Abstract of Short Communications, p. 86.

15. On structural stability. Annals of Math. Vol. 69, 1959, pp. 199-222.

16. Some examples on n-dimensional structural stability. Proc. Nat. Acad. Sci. Vol. 45, 1959, pp. 633-636.

17. Structural stability in the plane with enlarged boundary conditions. Anais da Academia Brasileira de Ciências, Vol. 31, 1959, pp. 135-160. (in collaboration with M.C. Peixoto).

18. Structural stability on two-dimensional manifolds. Boletin de la Sociedad Matemática Mexicana, 1960, pp. 188-189.

19. Structural stability on two-dimensional manifolds. Topology, Vol. 1, 1962, pp. 101-120.

20. Sobre o problema fundamental da teoria das equacões diferenciais. Atas of the 3rd Colóquio Brasileiro de Matemática, pp. 190-194. Fortaleza, 1961.

21. Structural stability on two-dimensional manifolds - a further remark. Topology, Vol. 2, 1963, pp. 179 -180.

22. On an approximation theorem of Kupka and Smale. Journal of Differential Equations, Vol. 3, 1967, pp. 214 - 227.

23. Qualitative theory of differential equations and structural stability. Proceedings of the International Symposium of differential equations and dynamical systems, Puerto Rico, 1965. Academic Press 1967, pp. 469-480. J. Hale and J.P. La Salle, ed.

24. Structurally stable systems on open manifolds are never dense. Annals of Mathematics, Vol. 87, 1968, pp. 423 - 430. (in collaboration with C.C. Pugh).

25. On a generic theory of end point boundary value problems. Anais da Academia Brasileira de CiŘncias, Vol. 41, 1969 pp. 1-6.

26. Sobre a classificacão das equacões diferenciais. Atas of the 6th Colóquio Brasileiro de Matemática, pp. 15-17. São Paulo, 1970.

27. Sur la classification des équations différentielles. C.R. Acad. Sc. Paris, Vol. 272, 1971, pp. 262-265.

28. Teoria geométrica das equacões diferenciais, 75 páginas. 7th Colóquio Brasileiro de Matemática, 1971.

29. On the classification of flows on 2 - manifolds. Proceedings of the International Symposium of Dynamical Systems. Salvador, Bahia, 1971. Academic Press, 1973, pp. 389-419.

30. Dynamical systems. Proceedings of the International Symposium of Dynamical Systems. Salvador, Bahia, 1971. Academic Press, 1973. (Editor of book and author of introductory chapter).

31. There is a simple arc joining two Morse-Smale flows. Société Mathématique de France, Astérisque, Vol. 31, 1976, pp. 16-41. (in collaboration with S. Newhouse).

32. On bifurcations of dynamical systems. Proceedings of the International Congress of Mathematicians, Vol. 2, 1975, pp. 315-319, Vancouver. Invited lecture.

33. Generic properties of ordinary differential equations. Math. Assoc. of America Studies in Mathematics, Vol. 14 1977, pp. 52-92, J. Hale, ed.

34. On end-point boundary value problems. Journal of Differential Equations, Vol. 44, 1982, pp. 273-280.

35. Le point de vue énumératif dans les problemes aux limites pour les équations différentielles ordinaires. I Quelques exemples. C.R. Acad. Sc. Paris, Vol. 303, Série I, number 13, 1986, pp. 629-633. Erratum, Vol. 307, 1988, pp. 197-198. (in collaboration with René Thom).

36. Le point de vue énumératif dans les problemes aux limites pour les équations différentielles ordinaires. II. Le théoreme. C.R. Acad. Sc. Paris, Vol. 303, Série I, number 14, 1986, pp. 693-698. (in collaboration with René Thom).

37. Acceptance speech for the TWAS 1986 Award in Mathematics (Beijing, 1987). From the volume: The future of Science in China and the Third World - Proceedings of

the Second General Conference Organized by the Third World Academy of Sciences, pp. 600-614. World Scientific, 1989.

38. Uma demonstracão do teorema do índice de Poincaré para superfícies. Matemática Universitária, Vol. 9/10, 1989, pp. 145 -151.

39. Enumerative two-point boundary value problems and a theorem of S. Bernstein. Anais da Academia Brasileira de Ciências, Vol. 62, 1990, pp. 321-327. (in collaboration with A.R. da Silva).

40. On the enumerative geometry of geodesics, in "From Topology to Computation- Proceedings of the Smalefest, Springer Verlag, ed. M.W. Hirsch, J. E. Marsden M. Shub, 1993, pp. 243-253. (in collaboration with I. Kupka).

41. "Some recollections on the early work of Steve Smale", in "From Topology to Computation- Proceedings of the Smalefest, Springer Verlag, ed. M. W. Hirsch, J. E. Marsden M. Shub, 1993, pp. 73-75. The above is a dinner speech delivered on August 6, 1990 at Berkeley on occasion of the cellebration of the sixtieth aniversary of Stephen Smale.

42. Sigma décomposition et arithmétique de quelques formes quadratiques définies positives. Published in the Festschrift de R. Thom: Passion des Formes. Editado por M. Porte, ENS Editions Fonteny-St.Cloud, 1994, pp. 455-479.

43. Focal decomposition in Geometry, Arithmetic and Physics. In Geometry, Topology and Physics. Eds. Apanasov / Bradlow / Rodrigues / Uhlenbeck, Berlin, Walter de Gruyter, 1997, pp. 213-232.