In accordance with the main objectives established for this project, during the duration of the project (2003-2006), characterisation and decidability problems in various algebraic and logical structures were studied. These studies were developed within the ambit of the four tasks identified.

2003

Decidability problems on finite semigroups

In this task we proved the tameness of **LSI**, the pseudovariety of local semilattices. We also proved that tameness is preserved under joins with the pseudovariety **K** of semigroups in which idempotents are left zeros. This last result was extended to other pseudovarieties, namely subpseudovarieties of the pseudovariety of J-trivial semigroups, and to the notion of complete tameness, a property which implies tameness.

Matrix Completion Problems

The N-matrix completion problem was studied based on the use of combinatorial methodology. The existence of an N-matrix completion of a given partial N-matrix has been guaranteed for combinatorially symmetric partial N-matrices whose associated graphs are 1-chordal or undirected cycles and for non-combinatorially symmetric partial N-matrices whose associated graphs are acyclic or cycles. The referred results have been organised and accepted for publication in Linear Algebra and its Applications.

Furthermore, triangular matrices over general rings were studied concerning generalized inverses. A joint work with R. Puystjens and R.E. Hartwig showed that these matrices are equivalent to a special ``orthogonal sum'', generalizing a previous result that focused a von Neumann regular completion of triangular matrices with regular diagonal components. The referred results have been organised and submitted for publication. Finally, the EP concept (introduced by Schwerdtfeger) was studied by means of a core-nilpotent decomposition, which led to an accepted paper in Linear Algebra and its Applications.

Proof Systems and Programming

The generalised multiary lambda-calculus *lambda J^m*, which extends lambda-calculus with a notion of generalised multiary application was defined. It was shown how *lambda J^m* captures naturally other extensions of lambda-calculus (namely, *Lambda J* of F. Joachimski and R. Matthes and *Lambda Ph* of J. Espírito Santo) and identified a set of permutative conversions on *lambda J^m* for which was shown confluence, termination and the permutability theorem. These ideas were presented in the International Conference TLCA. Confluence and strong normalisation of the notion of reduction were also established and their proofs are included in a recently submitted paper.

In collaboration with G. Barthe (INRIA Shopia Antipolis) and M.J. Frade (Informatics Department, University of Minho), L. Pinto, continued work on constructor subtyping systems. In particular, they established decidability of type-checking for the constructive subtyping core system and they considered a framework for extensible datatypes with overloading of constructors and overloaded extensive recursive functions, providing mechanisms to define recursive functions in systems of constructor subtyping with extensible datatypes. A paper with these ideas is under preparation.

Structure and order properties of semigroups

In accordance with the description of this task, the following results were obtained:

- Two characterisations of *F*-regular semigroups were obtained: one as regular monoids such that for every element *a* there exists a greatest *x* such that *ax* is an idempotent; the second as *E*-unitary regular monoids such that for every element *a* there exists a greatest *x* such that $axa \leq a$. The results obtained have been organised and accepted for publication in Journal of Algebra.

- The class of unary semigroups with an associate subgroup whose identity element is a medial idempotent was characterised. The special cases of completely regular and completely simple semigroups were also

characterised. The results obtained have been organised and submitted for publication.

2004

Decidability problems on finite semigroups

In collaboration with J. Almeida and M. Zeitoun, J.C. Costa obtained several decidability results for pseudovariety joins involving the pseudovarieties **J** and **R** of all J-trivial and R-trivial semigroups respectively. In particular, it was shown that the pseudovariety **V**∨**W** is (completely) tame when **V** is a subpseudovariety of **J** with decidable word problem and **W** is (completely) tame. Moreover **R**∨**W** is tame for a tame subpseudovariety **W** of the pseudovariety of all completely regular semigroups. These results are included in a paper submitted for publication. As an extension of the above results, the complete tameness of **R** was recently proved. A paper with this proof is being prepared.

Matrix Completion Problems

During this last year we have considered some classes of partial matrices, such as *N*-matrices and doubly negative matrices, and the related matrix completion problems, studied based on the use of combinatorial methodology. The existence of an *N*-matrix completion of a given partial *N*-matrix has been guaranteed for non-combinatorially symmetric partial *N*-matrices whose associated graphs are acyclic graphs or cycles, as well as some particular types of graphs in which cycles play an important role. The results obtained have been organised and published in Linear Algebra and its Applications. The *N*-matrix completion problem has also been considered under symmetry assumptions. It has been prooved that every partial doubly negative matrix whose associated graph is a *p*-chordal graph *G* has a doubly negative matrix completion if and only if $p = 1$. Furthermore, the question of completability of partial doubly negative matrices whose associated graphs are cycles has also been addressed. The obtained results have been organised and accepted for publication in Linear Algebra and

its Applications. In November 2004 the Ph.D. thesis of Cláudia Araújo, which presents research results in matrix completion problems, was approved.

Concerning matrices over arbitrary rings, we have shown that a lower triangular matrix, with a regular diagonal, is equivalent to its diagonal if and only if the matrix admits a lower triangular von Neumann inverse. This was proved by presenting a class of sufficient conditions to ensure that the sum $a + b$ in a ring R is equivalent to a sum $x + y$, which is an orthogonal Pierce decomposition. Applications include Roth decompositions of nxn block matrices, and related completion problems.

Proof Systems and Programming

This task continued the study of the generalised multiary lambda-calculus Lambda-J^m, an extension of Lambda-calculus with a notion of generalised multiary application. Confluence and strong normalisation of the various notions of reduction present in Lambda- J^m were established. These results were published in the post-proceedings of the Workshop TYPES 2004. A survey paper on Lambda- J^m is under preparation. Currently research on Lambda- J^m pursues two goals: (i) the study of the interaction between permutation rules and reduction rules; (ii) a refined analysis of the calculus, suggested by the overlapping between generality and multiarity.

Structure and order properties of semigroups

In accordance with the description of this task, the following results were obtained:

- Two characterisations of generalized F-regular semigroups were obtained: one as a semigroup that contains an anticone which is a principal order ideal of $(S, \leq_S)$; the second by means of the structure of the set of idempotents of S. The generalized F-semigroups in important classes of semigroups were also described. The results obtained have been organised and accepted for publication in *Mathematica Bohemia*.

- The description of all congruences on the semigroup generated by the nilpotent partial 1-1 transformations on an infinite set was obtained. A paper containing these results is under preparation.
- The homomorphisms between unary semigroups with an associate subgroup whose identity element is a medial idempotent were characterised. A paper containing the results obtained is under preparation.

2005

Decidability problems on finite semigroups

This task continued the study of the pseudovariety R of all R-trivial semigroups, in collaboration with J. Almeida (CMUP) and M. Zeitoun (LIAFA). This pseudovariety was proved to be completely tame and to have decidable pointlike sets. A survey paper about complete tameness in general, containing a sketch of the proof that R has that property (the complete proofs will be included in a paper which is in final preparation), has been submitted. A paper with the results concerning pointlikes for R is also being prepared.

Matrix completion problems

During 2006, and by joint work with Robert E. Hartwig, we finished and submitted a paper, entitled "On Roth's pseudo equivalence over rings", where we characterize the pseudo-equivalence of a block lower triangular matrix over a regular ring, and its block diagonal matrix, in terms of suitable Roth consistency conditions. The latter can in turn be expressed in terms of the solvability of certain matrix equations. Also, due to a joint work with R.E. Hartwig, studies were made in the direction of Drazin invertibility of a special sum. What followed generalize some known results of the literature. These were compiled into a paper and will be submitted, after revised.

Furthermore, during this last year we have considered some classes of partial matrices, such as totally nonpositive matrices and N_0-matrices, and the related

matrix completion problems, studied based on the use of combinatorial methodology. The existence of a totally nonpositive matrix completion of a given partial totally nonpositive matrix whose associated graph is a chordal graph or an undirected cycle was analysed. The obtained results have been organised and accepted for publication in Linear Algebra and its Applications. In what the N_0-matrix completion problem is concerned, some partial results were obtained on the study of the existence of the desired completion under some assumptions on the associated graph.

Proof systems and programming

J. Espírito Santo and L. Pinto continued work on the generalised multiary lambda-calculus Lambda- J^m. On the one hand, they studied the relationship of Lambda- J^m with natural deduction, found a new argument for strong normalisation of reduction and established preliminary results on the interaction between reduction and permutative conversions. On the other hand, jointly with M.J. Frade (Departamento de Informática, Universidade do Minho), they explored the overlap between generality and multiarity and produced a refined study of permutative conversions and a refined view of the internal structure of Lambda- J^m.

Structure and order properties of semigroups

A semigroup S is called F-monoid if S has an identity and if there exists a group congruence ρ on S such that each ρ-class $\leq_S$ The following characterizations of such monoids S were *established:* (i) by residuation of the identity, (ii) by the subsets $T(a)=\{x \in S: axa \leq_S a\}$, $a \in S$, (iii) by the maximal elements of $(S, \leq_S)$ and (iv) by means of an additional unary operation satisfying certain axioms. Three unary operations *, o and – on S defined by means of the greatest elements in the different ρ-classes of S were studied. Using their properties, a charaterisation of F-monoids S by their regular part $S^0 = \{a^0: a \in S\}$ and the associates of elements in S^0 was given. Under the hypothesis that $S^*= \{a^*: a \in S\}$ is a subsemigroup, it was also shown that S is regular. The results obtained have

been presented in a paper which has been accepted for publication in Communications in Algebra.

The study of the class of unary semigroups with an associate subgroup, whose identity element is a medial idempotent, was continued with the characterization of the lattice of congruences on semigroups of such class.

2006

Decidability problems on finite semigroups

This task continued the study of algorithmic properties in pseudovarieties of semigroups. Particular attention was devoted to the pseudovarieties *R*, of all *R*-trivial semigroups, and *LSl*, of all local semilattices. In collaboration with J. Almeida (CMUP) and M. Zeitoun (LaBRI), two papers were submitted concerning the pseudovariety *R*. The first one with a proof of its complete tameness, and the second one proving the decidability of its pointlike sets. In collaboration with C. Nogueira (ESTG-IPL), a paper was submitted where an infinite basis of identities for the omega-variety generated by the pseudovariety *LSl* is exhibited.

Matrix completion problems

A sign pattern matrix is a matrix with entries in the set {+,-,0}. Given a real matrix *A*, we define sign(*A*) as the sign pattern matrix obtained by replacing each negative entry of *A* by - and each positive entry by +. We say that a sign pattern matrix *P* is admissible for some class of real matrices if there exists a matrix *A* in that class with sign(*A*)=*P*. Some work was done in the characterization of the admissible sign pattern matrices of special classes of real matrices as *P*-matrices, *M*-matrices, inverse *M*-matrices and *N*-matrices. The results are being compiled and will be submitted for publication.

Further research was made concerning Drazin invertibility of a sum of ring elements with sertain properties, and its connection to the (Dedekind) finitness of the ring. These results were complied and will be submitted in the near future. This was a joint work with R.E. Hartwig. With respect to positive matrices, work

was developed on the caracterization of power-bounded matrices. This was a joint work with R.E. Hartwig.

Proof systems and programming

J. Espírito Santo and L. Pinto continued work on the generalised multiary lambda-calculus Lambda J^m. Jointly with M.J. Frade, they explored the overlap between generality and multiarity and produced a refined study of permutative conversions, a refined view of the internal structure of Lambda J^m and a first study of combined normal forms, by which important classes of normal forms were recovered.

J. Espírito Santo obtained a proof of strong cut-elimination by means of an embedding into the simply-typed lambda-calculus enriched with two permutative conversions.

In collaboration with T. Uustalu, L. Pinto worked on propositional bi-intuitionistic logic, aiming at establishing decision procedures and methods for counter-model construction.

In collaboration with R. Matthes, J. Espírito Santo and L. Pinto defined continuation-passing style translations for various intuitionistic extensions of simply-typed lambda-calculus, obtaining simulation of reduction and therefore syntactic proofs of strong normalisation.

Structure and order properties of semigroups

Let S be the model of a regular semigroup with an associate subgroup whose identity is a medial idempotent. For another such semigroup T, we constructed all unary homomorphisms of S into T in terms of their parameters. On S we constructed all unary congruences again directly from its parameters. This construction leads to a characterization of congruences in terms of kernels and traces. We described the K-, T-, T_1-, K_1-, L-, U- and V-relations on the lattice of all unary congruences on S, again in terms of parameters of S. The results obtained have been organized and submitted for publication.

Based on the work published in 1999 on the ideal structure of nilpotent-generated transformation semigroups, the lattice of congruences on both the semigroup generated by injective nilpotent transformations and the semigroup generated by partial nilpotent transformations were described. The results obtained have been presented in two papers, one of which has been published in Bull. Austral. Math. Soc. and the other one submitted for publication.