FINITENESS AND RATIONAL DATATYPES, CONSTRUCTIVELY

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In constructive logic, finiteness of a set [or of a subset of a given set] can be defined in several inequivalent ways, and there is no obvious "right" definition. The situation is especially subtle, if equality on the set [or equality on the encompassing set] is not decidable [or the predicate defining the subset is not decidable]. Two of the most fundamental notions of finiteness are listability and Noetherianness, listability being generally stronger.

For a given a branching type, rational trees are by definition those non-wellfounded trees that have a finite number of distinct subtrees, extensional equality between non-wellfounded trees being given by bisimilarity. Since this definition refers to finiteness, different notions of finiteness could a priori lead to different notions of rationality.

In this talk, I explain the relationship between different notions of finiteness generally and in the special case of subsequences of a given sequence. I demonstrate that, for subsequences of a sequence, listability and Noetherianness are equivalent and exactly this equivalence leads to useful function definition and reasoning principles for rational sequences, including an inductive representation. Similar considerations apply to rational datatypes generally.

We are formalizing this development in the dependently typed programming language Agda.

This is ongoing joint work with James Chapman and Niccolò Veltri.

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