Radiative gravitational collapse to spherical, toroidal and higher genus black holes

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December 2016

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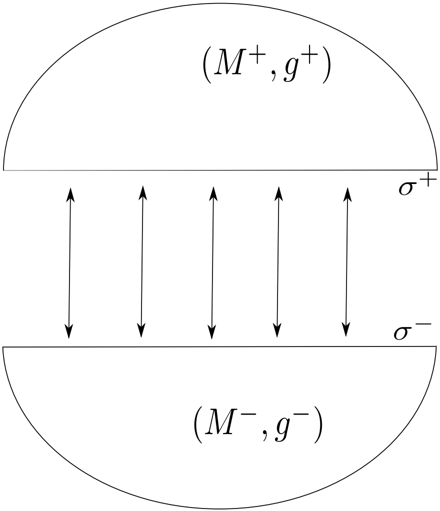
- AdS/CFT correspondence.
- Local topologies can be different from asymptotically far topologies.
- Many possible topologies in more than 4 dimensions.

Spacetime Matching

We consider two spacetimes $(M^{\pm}, g^{\pm})$ with boundaries $\sigma^{\pm}$.



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$$K^{+}_{ij} \overset{\sigma}{=} K^{-}_{ij}$$

These conditions prevent infinite discontinuities of matter and curvature across the hypersurface.

Spacetimes to be Matched - Friedmann-Lemaître-Robertson-Walker (FLRW)

FLRW Metric

$$ds^2 = -dt^2 + a^2(t)\Big(dR^2 + f^2(R)\big(dx^2 + g^2(x)\,dy^2\big)\Big)$$

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$k = 1$	$k = 0$	$k = -1$
$f = \sin R, \cos R$; $g = \sin x, \cos x$	(a) $f = 1 = g$ (b) $f = R;\ g = \sin x, \cos x$	(a) $f = e^{\pm R};\ g = 1$ (b) $f = \sinh R;\ g = \sin x, \cos x$ (c) $f = \cosh R;\ g = \sinh x, \cosh x$

- All $k = 1$ cases correspond to **spherical** geometry.
- For $k = 0$ or $k = -1$: (a) **planar** geometry, (b) **spherical** geometry and (c) **hyperbolic** geometry.

Spacetimes to be Matched - Robinson-Trautman

Robinson-Trautman Metric

$$ds^2 = -\chi du^2 + 2\varepsilon du dr + r^2(d\theta^2 + \Sigma^2(\theta) d\phi^2)$$

$$\chi = b - \frac{2m(u)}{r} - \frac{\Lambda}{3} r^2$$

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- u is a **null advanced** ($\varepsilon = +1$) or **retarded** ($\varepsilon = -1$) coordinate.
- $\Sigma(\theta) = \theta, \sin\theta$ or $\sinh\theta$ and $b = 0, 1, -1$ for **planar**, **spherical** or **hyperbolic** geometry, respectively.

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$$T_{\alpha\beta} = \rho(u, r) k_\alpha k_\beta \qquad\qquad k = -du$$

The Schwarzschild metric is obtained for constant m, $\Lambda = 0$ and $b = 1$.

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$$\begin{aligned}
\dot{u} &\overset{\sigma}{=} \varepsilon \frac{\dot{t} + \varepsilon\bar{\epsilon}\epsilon a \dot{R}}{\varepsilon\bar{\epsilon}\epsilon f' - a_{,t} f} \\
r &\overset{\sigma}{=} af \\
m &\overset{\sigma}{=} \frac{a^3 f^3 \rho}{6} \\
\dot{R} &\overset{\sigma}{=} \varepsilon\bar{\epsilon}\epsilon \frac{p}{a\rho} \dot{t} \\
\Sigma(\vartheta) &\overset{\sigma}{=} g(\vartheta)
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We will choose a linear equation of state $p = \gamma\rho$.

Results

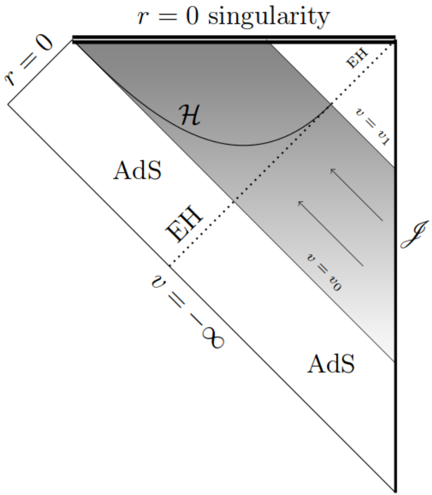

Figure : Black hole formation in toroidal AdS space through ingoing radiation. Source: Senovilla J. M. M., "Black hole formation by incoming electromagnetic radiation", *Class. Quantum Grav.*, **32** (2015) 017001.

Results for $\Lambda < 0$

Spherical Topology with $k = 0$ and $b = 1 \quad \Rightarrow \quad f(R) = R$.
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	$\varepsilon = 1$	$\varepsilon = -1$
$\bar{\epsilon}\epsilon = 1$	Initially untrapped Black Hole formation	Initially trapped White Hole evaporation
$\bar{\epsilon}\epsilon = -1$	Initially trapped Black Hole	Initially trapped White Hole

- $\varepsilon = 1$: start with a **finite** m which **diverges** as the matching hypersurface approaches the singularity.
- $\varepsilon = -1$: start with an **infinite** m which **decreases** to finite over time.

Case $\overline{\epsilon\epsilon} = -\varepsilon = 1$ is the time reversal of the case $\overline{\epsilon\epsilon} = \varepsilon = 1$.

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For $R(\tau = 0) = 0$:

- we can only have $\varepsilon = \bar{\epsilon}\epsilon = 1$ and a **zero mass** m (AdS spacetime) which then **diverges** towards infinity.

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Toroidal topology with $b = 0$ and $k = -1 \quad \Rightarrow \quad f = e^{\pm R}$.

- $\gamma = 1/3$: Black Hole formation is possible for $\varepsilon = 1$ and $\bar{\epsilon}\epsilon = \pm 1$ for $f = e^{\pm R}$ respectively. The mass m starts as **finite** and **positive**, **diverging** as the hypersurface approaches the singularity.
- $\gamma = -2/3$: White Hole evaporation is possible for $\varepsilon = -1$, $\bar{\epsilon}\epsilon = 1$. The mass m starts as **finite** and **decreases** to zero.

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Higher Genus topology with $b = k = -1 \quad \Rightarrow \quad f = \cosh R$

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The End

Thanks for your presence!